ICDEA 2021

International Society of Difference Equations

ISDE

26th International Conference on Difference Equations and Applications

26-30 July, 2021, Sarajevo, Bosnia and Herzegovina

ABSTRACTS BOOK

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ODSJEK ZA MATEMATIKU

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ICDEA 2021

The 26th International Conference
on Difference Equations and Applications
University of Sarajevo

26-30 July, 2021

Sarajevo, Bosnia and Herzegovina

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Welcome message

Dear Colleagues

We look forward to welcoming you to the first-ever virtual conference on difference equations and applications, "The 26th International Conference on Difference Equations and Applications (ICDEA 2021)". While we regret that the COVID-19 pandemic prevented us from holding the meeting in Sarajevo, we are excited about the opportunity of having a virtual conference. Furthermore, having the conference online allows reaching broader participants than a traditional in-person conference could include.

A total of 258 participants registered for this conference from all over the world. The conference features 14 plenary speakers experts in difference equations/discrete dynamical systems and their interplay with nonlinear sciences. Furthermore, five special sessions cover various themes in difference equations/discrete dynamical systems and their applications. In addition, there are 83 contributed talks on various topics in difference equations/discrete dynamical systems and their applications in biology, economy, engineering, game theory, social sciences, etc.

We are sure that this conference will be motivating, significant, and valuable for all of you.

Welcome to ICDEA 2021 Virtual!

Organizing Committee of ICDEA 2021

About Sarajevo

Sarajevo city is the capital of Bosnia and Herzegovina, situated on the Miljacka River, and it has always been smack bang on a geopolitical fault line. During the Roman Empire, Sarajevo, together with Bosnia, was a border city between the Eastern and Western Roman Empires.

In the Middle Ages, the city had the name Vrh-Bosna until it fell under the control of the Ottoman Empire in 1429 and was renamed Bosna-Saraj or Bosna-Seraj. During the Berlin Congress in 1878, Sarajevo was taken from the Ottomans and given to the Austro-Hungarian Empire, again right on the borderline between two Empires, between East and West, between Islam and Christianity the last 100 years, Sarajevo has been a member of six different states. On the 28th of June 1914, WWI was triggered by the assassination of Archduke Franz Ferdinand of Austria with his wife Sophie, Duchess of Hohenberg.

The city has ten bridges over the Miljacka River. The most famous one is the Latin Bridge or Princip Bridge, the name of the assassin of Archduke Franz Ferdinand. The bridge is on the coat of arms of Sarajevo. The city-wide tram service was the very first in Europe. Locals proudly insist that the Austro-Hungarians modeled Vienna's tram system on theirs. The first Winter Olympic Games in Communist country were held in Sarajevo in February 1984, winning over Sapporo, Japan, and Gothenburg, Sweden.

Sarajevo had the longest-running siege of any town in modern war history (1425 days). Sarajevo was, and still is, a very culturally diverse city proudly known as the European Jerusalem – within a very short walking distance, you come across Orthodox and Catholic churches, synagogues, and mosques.

Plenary Talks

Plenary Speakers

John Appleby, Dublin City University, Ireland

Francisco Balibrea, Universidad de Murcia, Spain

Jernej Činč, University of Vienna, Austria and IT4Innovations, Ostrava, Czech Republic

Jim M. Cushing, The University of Arizona, USA

Saber Elaydi, Trinity University, USA

Peter E. Kloeden, University of Tubingen, Germany

René Lozi, Université de Nice Sophia-Antipolis, France

Nina Snigireva, University College Dublin, Ireland

Lubomir Snoha, Matej Bel University, Banska Bystrica, Slovakia

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Mean Square Characterisation of Linear Stochastic Equations with Memory

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Emmet Lawless

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Presentation type: Plenary Talk

Necessary and sufficient conditions for the asymptotic stability of deterministic linear autonomous equations with memory have been long understood. However, the situation for stochastic equations with memory, whether in continuous or discrete time, is not so well settled. In this talk, we concentrate on the scalar case (in discrete and continuous time) and on the asymptotic behaviour of the mean square of the solution.

It is a ready consequence of work in the literature that, in the case of difference equations with finite memory, ideas laid out by Bellman can be used to resolve the matter completely. In the scalar case, however, this comes at the cost of high–dimensional conditions. Moreover, these ideas do not seem to transfer so smoothly to Volterra equations (which have unbounded memory), or to continuous equations.

Our approach here is to create an auxiliary summation (or integral) equation, and to determine the asymptotic behaviour using renewal theorems. This approach characterises the asymptotic behaviour of the original stochastic equation in terms of a functional of the fundamental solution of the underlying deterministic equation which has been stochastically perturbed. A formula for this functional can always be found in terms of the problem data, and in some cases it can be computed in closed–form. Moreover, the discrete–time calculations also guide us how to proceed in the continuous case. In particular, one can provide a closed–form characterisation of the mean square behaviour of scalar stochastic delay differential equations with a single delay, achieving parity with the deterministic case. This is satisfactory, since the deterministic case has been understood since the 1950s.

Lastly, this work shows that the asymptotic behaviour in mean square gives rise to new problems which are principally deterministic in character, and which can perhaps be solved using existing non–stochastic methods.

HOMOCLINIC TRAJECTORIES IN DYNAMICAL SYSTEMS OF LOW DIMENSION

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Presentation type: Plenary Talk

On a dynamical system (I, f) where I is a compact real interval and $f \in C(I, I)$, a *homoclinic trajectory* is a non-periodic trajectory whose $\alpha - limit$ and $\omega - limit$ sets coincide and are a cycle (a periodic trajectory). A point is called *homoclinic* if it belongs, at least one, homoclinic trajectory.

The existence of a homoclinic trajectory usually indicates that in the system there are trajectories of very complicated dynamical behaviour.

(I, f) has a homoclinic trajectory if and only if the map has a cycle whose period is not a power of two (Block-Coppel book and Fedorenko-Sharkovsky). Also both conditions are equivalent to positive topological entropy.

Homoclinic trajectories on dynamical systems on the square $Q = [0, 1]^2$ can also be considered using the same definitions than in the interval case. It is a more difficult case to construct them. We will consider *triangular maps* of the form

$$F(x, y) = (f(x), g(x, y))$$

where f and g are real continuous maps in their domais of definition, respectively I and I^2.

Jointly with J.Smítal we have constructed an example of a map in the above family possesing a homoclinic trajectory $f(x) = \begin{cases} 3x & \text{if} x \in [0, \frac{1}{3}], \\ 1 & \text{if} x \in [\frac{1}{3}, \frac{2}{3}], \\ 3(1-x) & \text{if} x \in [\frac{2}{3}, 1] \end{cases}$

Then f has periodic trajectories of all periods since has a periodic trajectory of period 3. Also it is not difficult to construct a homoclinic trajectory to the fixed point 0, that given by

$$\Gamma_0 = \{0\} \cup (3^{-n})_{n=0}^{\infty}$$

since $f(1) = 0$ and $f(3^{-n-1}) = 3^{-n}$ when $n \geq 0$.

We will consider other two dimensional systems on the square out of the family of triangular maps where it is difficult to give explicit expressions of homoclinic trajectories. In the first example it is studied also the dependence of such trajectories on parameter a. In the case of diffeomorphisms we have to introduce new adapted definitions

Difference Equations and Darwinian Dynamics

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Presentation type: Plenary Talk

Although difference equations have a long and illustrious history of application to population dynamics, virtually none of these equations take evolutionary change by natural selection, arguably the most fundamental principle in biology, explicitly into account. I will describe a modeling methodology (Darwinian dynamics or evolutionary game theory) by means of which evolutionary adaptation can be included in any difference equation model for the dynamics of a biological population. I will discuss some basic theorems that address the fundamental biological question of extinction versus survival and illustrate the methodology with applications to a couple of questions of contemporary questions concerning evolutionary adaptation. My primary goal is to introduce these types of equations to researchers in difference equations for which there are many interesting challenges, open questions, and opportunities to contribute mathematically and biologically to this field.

Dynamical planar embeddings of tent inverse limit spaces.

Jernej Činč

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(joint work with Ana Anušić (University of São Paulo) and Henk Bruin (University of Vienna))
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Presentation type: Plenary Talk

Brown-Barge-Martin (BBM) embeddings of inverse limit of a parameterised family of tent maps yield a natural way to construct a parameterised family of strange attractors arising from planar homeomorphisms. This family of inverse limits has seen much attention in the past three decades due to the classification problem known as the Ingram conjecture. After Barge, Bruin and Štimac [3] resolved the conjecture, this family has seen another surge in activity related with the detailed study of BBM embeddings of tent inverse limit spaces which culminated in the work of Boyland, de Carvalho and Hall [4] and Anušić and Činč [2]. In this talk I will overview two approaches for studying BBM embeddings and discuss how Lorenz interval maps and Sturmian sequences appear naturally in this study, see Anušić, Bruin and Činč [1]. I will close the talk with some remaining questions about (non)-extendability of the natural extensions of inverse limits of tent maps and about the topological structure of tent inverse limits. This talk is based on the joint works with Ana Anušić (University of São Paulo) and Henk Bruin (University of Vienna).

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Non-autonomous difference equations/discrete dynamical systems and applications

Saber Elaydi

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Presentation type: Plenary Talk

In this talk, we will give a survey on some of the developments in non-autonomous discrete systems generated by infinitely many continuous maps defined on locally compact metric spaces. The focus will be on non-autonomous systems that are asymptotic to autonomous systems or periodic systems.

Applications to evolutionary biological models will be investigated. Further applications to some epidemic models will be discussed.

At the end of the talk, we will present some open problems.

Lattice difference equations

Peter E. Kloeden

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Presentation type: Plenary Talk

Lattice difference equations are essentially difference equations on a Hilbert space of bi-infinite sequences. They are motivated by the discretisation of the spatial variable in integrodifference equations arising in theoretical ecology. It is shown here that under similar assumptions to those used for such integrodifference equations they have a global attractor, to which the global attractors of finite dimensional approximations converge upper semi continuously. Corresponding results are also shown for lattice difference equations when only a finite number of interconnection weights are nonzero and when the interconnection weights themselves vary and converge in an appropriate manner. Random lattice difference equations and their random attractors will also be discussed.

Researches on actual or industrial applications of difference equations: a chimeric task?

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Presentation type: Plenary Talk

Refering to the 26 issues of Journal of Difference Equation and Applications published between 1995 and 2020, among the 2035 published research articles only 161 are dedicated to applications in a very broad sense (8%). During the 10 first years only 12 over 491 articles (2.4%) belong to this category of papers. This percentage is slowly increasing since, in the next 10 issues the ratio is 56/956 (5.9%) and finally in the last six issues it climbs to 93/588 (10.1%). In their inaugural editorial Saber Elaydi and Gerry Ladas concluded "In addition, applications are seen in such diverse disciplines as chaos theory, fractal theory, population dynamics, public health, game theory, operations research, statistics, sociology, economics, control theory, combinatorics, and numerical simulations of complex systems"[1]. Does this goal is fully reached 26 years after? The analysis of the topics covered in these 161 articles can be done splitting them in four categories:

Public health, biology, and population: 98 articles (61%) [modeling the spread of diseases, parasites, SIR model: 43 articles (26.7%); population dynamics: 37 articles (23%); Predator-Prey model: 14 article (8.7%); Neurons: 4 articles (2.5%)]

Economy money, market, etc.: 29 articles (18%)

Technical: 27 articles (16.8%) [Physics and Chemistry: 16 articles (10%); Cryptography, big data, signal processing, random generator: 8 articles (5%); Optimization, control: 3 articles: (1.9%)]

Miscellaneous: 7 articles (4.3%).

Therefore, if population dynamics, public health and economics are well represented with nearly four fifth of the publications, application of chaos theory, game theory, operation research, statistics, sociology, control theory, cryptography, simulation of complex systems, received a poor attention from potential authors. We will try to understand what are the hidden reasons of such lack of interest for application of difference equations in these topics (competition with differential equations, with other scientific journals, etc.?) and how to improve the interest for such actual or industrial applications.

References

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Beyond contractive iterated function systems

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Presentation type: Plenary Talk

In this talk we will give an overview of weakly contractive iterated function systems (IFS) and highlight some results which prove useful when analysing noncontractive IFSs. For example, we will discuss the Lasota-Myjak theory of semiattractors and show how it can be used to explain the behaviour of certain noncontractive IFSs. We will then consider examples of noncontractive IFSs which admit semiattractors. In particular, we will focus on IFSs enriched with isometries. We will discuss the chaos game algorithm for such systems.

Generic chaos

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Presentation type: Plenary Talk

A topological dynamical system (X, f) given by a continuous selfmap f of a metric space X is called *generically chaotic* or *generically ε-chaotic* if the set of all scrambled pairs or ε-scrambled pairs is residual in X^2, respectively.

The notion of generic chaos was suggested by A. Lasota. It is well understood on the interval, mainly because generic chaos on the interval is equivalent with generic ε-chaos and the latter can be characterized in terms of the behaviour of open balls under iterations. Such a characterization of generic ε-chaos works in a large class of metric spaces, therefore for the study of generic chaos it is important to know in which spaces, besides the interval, generic chaos is equivalent with generic ε-chaos. Indeed, in such spaces we can check generic chaos 'macroscopically', in terms of the behaviour of open balls (to verify generic chaos using just the 'microscopic' definition appears to be an almost impossible task). It has been known that all graphs are such spaces, but not all dendrites.

In the first part of the lecture we will discuss some properties of scrambled pairs and scrambled sets and some known facts on generic chaos. Then we will describe the full topological characterization of those dendrites, on which generic chaos is equivalent with generic ε-chaos, i.e. dendrites on which generic chaos can be verified 'macroscopically'.

References

[1] Ľ. Snoha, V. Špitalský and M. Takács, *Generic chaos on dendrites*, Ergodic Theory Dynam. Systems, published online 19 March 2021, 43 pp.; arXiv:2102.00486 [math.DS]

Discrete Painlevé equations and recurrence coefficients for orthogonal polynomials

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Presentation type: Plenary Talk

Orthogonal polynomials on the real line always satisfy a three term recurrence relation. The recurrence coefficients are explicitly known for classical orthogonal polynomials, but for semiclassical orthogonal polynomials these recurrence coefficients satisfy (a system of) non-linear recurrence relations, which can be identified as discrete Painlevé equations. I will present various examples of this and discuss how these are related to the Painlevé differential equations. In many cases one needs a special solution of these discrete Painlevé equations satisfying some constraints.

References

[1] W. Van Assche, *Orthogonal Polynomials and Painlevé Equations*, Australian Mathematical Society Lecture Series **27**, Cambridge University Press, 2018.

A Derivation Procedure and Analysis of Two-Species Difference Equation Population Models

Gail Wolkowicz

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Presentation type: Plenary Talk

A derivation procedure different from the commonly applied Euler discretization scheme is proposed to formulate discrete two species population models. By distinguishing growth and decline processes, we determine a multiplicative net per capita growth ratio. This technique is first applied to predator-prey relations. Under the assumption that the predator declines exponentially in the absence of the prey and that the prey grows logistically in the absence of the predator, a discretization of the classical Lotka–Volterra predator-prey model is obtained. Applying the same derivation method to two competing species results in a competition model, that, in its simplest form, is sometimes referred to as the discrete Leslie/Gower model [1]. The analysis of these models shows the usefulness of nullclines in combination with ther associated *root-functions*. By determining the uniqueness of these positive root-functions and their position in the first quadrant, we present an elementary approach to study the global dynamics of non-negative equilibria. Applying this technique to the two-species competition model derived, we study the global stability of the non-negative equilibria and offer an alternative method to the one used in [1, 2, 3]. For the predator-prey model derived, the root-functions and nullclines are used to study the global stability of the boundary equilibria, as well as, to prove the non-existence of n-periodic orbits for $n = 2, 3$.

References

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Tiny models can help us understand huge models

James A. Yorke

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Presentation type: Plenary Talk

I will be discussing a couple related “simple” maps that tell us a great deal about very complex situations. This is joint work with Roberto De Leo, Yoshi Saiki, Shuddho Das, Miguel Sanjuan, and Hiroki Takahasi. I will report mainly on 3 of our papers papers that have been accepted for publication this year and are available on arXiv. Two are on the logistic map $rx(1-x)$. It has uncountably many parameter values with the following property: The map has infinitely many disjoint unstable compact invariant Cantor sets. We have determined how their stable and unstable manifolds interact. All this in a logistic map. Secondly, we have created a baker-like piecewise linear map on a 3D cube that is unstable in 2 dimensions in some places and unstable in 1 in others. It has a dense set of periodic orbits that are 1D unstable and another dense set of periodic orbits that are all 2D unstable. The map is ergodic. Lebesgue measure is invariant.

A Periodic Discrete Dynamical Model on Wolbachia Infection Frequency in Mosquito Populations

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Presentation type: Plenary Talk

How to prevent and control the outbreak of mosquito-borne diseases, such as malaria, dengue fever and Zika, is an urgent worldwide public health problem. The most conventional method for the control of these diseases is to directly kill mosquitoes by spraying insecticides or removing their breeding sites. However, the traditional method is not effective enough to keep the mosquito density below the epidemic risk threshold. With promising results internationally, the World Mosquito Program's *Wolbachia* method is helping to reduce the occurrence of diseases transmitted by mosquitoes. In this talk, we will introduce a generalized discrete model to study the dynamics of *Wolbachia* infection frequency in mosquito populations where infected mosquitoes are impulsively released. This generalized model covers all the relevant existing models since 1959 as some special cases. After summarizing known results of discrete models deduced from the generalized one, we put forward some interesting questions to be further investigated for the periodic impulsive releases.

References

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Invariant Manifolds with/without Hyperbolicity

Weinian Zhang

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Presentation type: Plenary Talk

In this talk some advances on invariant manifolds are introduced under assumptions of hyperbolicity, nonuniform hyperbolicity, pseudo-hyperbolicity, or no hyperbolicity. In order to describe hyperbolicity, related problems on roughness and admissibility for exponential dichotomies are discussed.

References

[1] Wenmeng Zhang and Weinian Zhang, *On invariant manifolds and invariant foliations without a spectral gap*, Advances in Math. **303** (2016), 549-610.

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Special Sessions

Special Sessions

Jim Cushing and Gail Wolkowicz: Population Dynamics and Related Topics

Laura Gardini and Irina Sushko: Invertible and Noninvertible Maps: Theory and Applications

Mustafa Kulenović: Global Dynamics of Monotone Discrete Dynamical Systems

Lubomir Snoha: Topological and Low-Dimensional Dynamics

Jianshe Yu, Jia Li and Bo Zheng: Nonlinear Difference Equations and their Applications in Biological Dynamics

Population Dynamics and Related Topics

Chairs: Jim Cushing and Gail Wolkowicz

List of Speakers

Azmy Ackleh, University of Louisiana at Lafayette, USA

Ziyad AlSharawi, Department of Mathematics and Statistics American University of Sharjah, UAE

Steve Baigent, Department of Mathematics, UCL, UK

Elena Braverman, Dept. of Mathematics and Statistics, University of Calgary, Canada

Daniel Franco, Department of Applied Mathematics, Universidad Nacional de Eduación a Distancia(UNED), Spain

Zhanyuan Hou, School of Computing and Digital Media, London Metropolitan University, UK

Senada Kalabušić, University of Sarajevo, Bosnia and Herzegovina

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Stability Results for Discrete-Time Predator-Prey Models

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Presentation type: Special Session Talk
The title of the special session: Population Dynamics and Related Topics

We consider a two-dimensional discrete-time predator-prey model that was developed by Ackleh et. al (2019). We derive conditions for the global stability of the unique interior equilibrium using an approach based on nullcline analysis. Then, we consider a threedimensional evolutionary counterpart developed in Ackleh et. al (2019) which couples the population dynamics with the dynamics of an evolving phenotypic trait. Using a perturbation argument we extend the global stability results to the interior equilibrium of the threedimensional predator-prey model. If time permits, we will present an extension of the twodimensional model to a three-dimensional predator-prey model with stage structure in the predator and study its local dynamics.

References

[1] A.S. Ackleh, Md I. Hossain, A. Veprauskas and A. Zhang, Persistence and stability analysis of discrete-time predator-prey models: A study of population and evolutionary dynamics, Journal of Difference Equations and Applications, 25(2019), 1568-1603.

The role of vigilance on a discrete-time predator-prey model

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Presentation type: Special Session Talk
The title of the special session: Population Dynamics and Related Topics

In this talk, we consider a discrete-time predator-prey model in which vigilance of prey act as a tradeoff between the safety and growth rate of the prey. We consider the vigilance parameter as the main parameter and investigate mathematical properties of the system such as stability, permanence, both period-doubling and Neimark-Sacker bifurcations of the model. Numerical simulations are carried out to illustrate the analytical findings and to further explore the impact of prey vigilance on the dynamics of the system. Finally, some mathematical questions that need further investigation will be posed.

References

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Global stability of population models

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Presentation type: Special Session Talk
The title of the special session: Population Dynamics and Related Topics.

I will discuss the 'split Lyapunov function' method (introduced for Lotka-Volterra dynamics in [1]) and its applications to establishing global stability of fixed points in finite dimensional Kolmogorov-type models of population growth [2]. Time permitting, I will comment on how the split Lyapunov function method links to the carrying simplex, an attracting invariant manifold codimension one that often appears in the models that I discuss.

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Global attractivity of compartment models and neural networks with a distributed delay

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Presentation type: Special Session Talk
The title of the special session: Population Dynamics and Related Topics.

We consider a system of nonlinear differential equations with a distributed delay

$$\frac{dx_i}{dt} = g_i(t)\left[\int\limits_{h_{i1}(t)}^{t} d_{\tau_1} r_{i1}(t,\tau_1) \cdots \int\limits_{h_{is}(t)}^{t} f_i(x_1(\tau_1), x_2(\tau_2), \dots, x_s(\tau_s))\, d_{\tau_s} r_{is}(t,\tau_s) - x_i(t)\right]$$

and obtain global asymptotic stability conditions, which are independent of delays. The ideas of the proofs are based on the notion of a strong attractor of a vector difference equation associated with a nonlinear vector differential equation. The results are applied to compartment-type models of population dynamics with Nicholson's blowflies growth law and to Hopfield neural networks. The ideas of the proofs are based on the notion of a strong attractor of a vector difference equation associated with a nonlinear vector differential equation. The results extend the theorem that for a one-dimensional equation with a distributed delay, delay-independent stability can be deduced from attractivity of an associated difference equation [1]. The talk describes the progress generalizing [2] recently published in [3].

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A new tool for studying the global stability of discrete-time population models

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Presentation type: Special Session Talk
The title of the special session: Population Dynamics and Related Topics.

In this talk, we will present a new geometric method to study the stability of one-dimensional discrete-time models. We will provide examples to illustrate how the method works. In particular, we will show that it can be used to complement and extend some stability results in the literature both for discrete-time models and continuous-time models with delay. The talk is based on work in collaboration with J. Perán, J. Segura, C. Guiver and H. Logemann [1, 2, 3].

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Modified carrying simplex and global dynamics of competitive maps

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Presentation type: Special Session Talk
The title of the special session: Population Dynamics and Related Topics

This talk is based on the author's recent paper [3]. For a C^1 map T from $C = [0, +\infty)^N$ to C of the form $T_i(x) = x_i f_i(x)$, the dynamical system $x(n) = T^n(x)$ as a population model is competitive if $\frac{\partial f_i}{\partial x_j} \leq 0$ $(i \neq j)$. A well know theorem for competitive systems, presented by Hirsch [2] and proved by Ruiz-Herrera [1] with various versions by others (such as [4]), states that, under certain conditions including the map being competitive (retrotone), the system has a compact invariant surface $\Sigma \subset C$ (called carrying simplex) that is homeomorphic to $\Delta^{N-1} = \{x \in C : x_1 + \cdots + x_N = 1\}$, and every trajectory in $C \setminus \{0\}$ is asymptotic to one in Σ. The theorem has been well accepted with applications requiring all the N^2 entries of the Jacobian matrix $Df = (\frac{\partial f_i}{\partial x_j})$ to be negative. We point out that, with a modified carrying simplex in mind, the above requirement is unnecessarily strong and too restrictive. We prove the existence and uniqueness of a modified carrying simplex by reducing that condition to requiring every entry of Df to be nonpositive and each f_i is strictly decreasing in x_i. As a example of applications of a modified carrying simplex, sufficient conditions are provided for vanishing species and dominance of one species over others. Another example is the global attraction (repulsion) of a heteroclinic cycle depending on the global repulsion (attraction) of the unique interior fixed point.

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Dynamics of a plant-herbivore models with logistic growth of plant biomass

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Presentation type: Special Session Talk
The title of the special session: Population Dynamics and Related Topics

We investigate the plant-herbivore model's dynamics. The plant's biomass without herbivores grows with a logistic equation assuming that the herbivore (parasitization) occurs after the host's density-dependent growth regulation occurs. We classify the equilibrium points. We show that the boundary equilibrium undergoes the transcritical, fold, and period-doubling bifurcation, whereas the interior equilibrium undergoes a Neimark-Sacker bifurcation. Furthermore, the system exhibits bistability between the stable interior attractors in the interior and the stable attractors in the $x-$boundary logistic dynamics (periodic orbits and strange attractors) for particular numerical values of parameters. Thus, sufficient conditions for the permanence of the plant-herbivores system are obtained, ensuring the coexistence of both species.

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Optimal control of a discrete-time plant-pest model with bistability and fluctuating environments

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Presentation type: Special Session Talk
The title of the special session: Population Dynamics and Related Topics

Discrete-time plant-pest models with two different constant control strategies (i.e., removal versus reduction strategies) have been investigated to understand how to regulate the population of pest. The corresponding optimal control problem has been explored on three scenarios of bistability plant-pest dynamics where these dynamics are determined by the growth rate of the plant and the damage rate inflicted by pest. Furthermore, the impacts of fluctuating environments on discrete-time plant-pest dynamics have been explored. Through analysis and simulations, we identify and evaluate the optimal controls and their impact on the plant-pest dynamics. There are critical factors to characterize the optimal controls and the corresponding plant-pest dynamics such as the control upper bound (the effectiveness level of the implementation of control measures) and the initial conditions of the plant and pest. The results show that the pest is hard to be eliminated when the control upper bound is not large enough or the initial conditions are chosen from the inner point of the basin of attractions. However, as the control upper bound is increased or the initial conditions are chosen from near the boundary of the basin of attractions, then the pest can be manageable regardless of fluctuating environments.

Discrete-time Models for Interactive Dynamics of Wild and Sterile Mosquitoes

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Presentation type: Special Session Talk
The title of the special session: Population Dynamics and Related Topics

In this talk, we present discrete-time models for interactive dynamics of wild and sterile mosquitoes. The survival functions are assumed to be of the Ricker-type or Beverton-Holt-type nonlinearity. We investigate the stability of the trivial fixed point and the existence and stability of positive fixed points. Other dynamical features, such as periodic solutions, are demonstrated as well. We show both theoretical and numerical results.

Linear stability conditions for a first order n-dimensional mapping

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Presentation type: Special Session Talk
The title of the special session: Population Dynamics and Related Topics

Stability is one of the most important concepts in dynamical systems. It is well known that, a fixed point of a discrete dynamical system is locally asymptotically stable if all the eigenvalues of the Jacobian matrix, evaluated at the fixed point, are less than one in absolute value.

However, in the most cases, we are not able to compute the eigenvalues. The challenge here is to provide stability conditions without knowing the eigenvalues of the Jacobian matrix.

The necessary and sufficient algebraic conditions for the roots of a real polynomial to lie inside the unit circle have been established by Jury [2] in a table form, where the constraints are obtained only by evaluation of second-order determinants.

In order to determine such a table, it is necessary to know all the coefficients of the characteristic polynomial of the Jacobian matrix. Those coefficients may be obtained from the trace, the determinant and the minors of the Jacobian matrix.

In this talk, I will present an alternative way to compute the values of the coefficients of a characteristic polynomial. I use some specific notation that turns such computation friendly, specifically from the computational point of view.

The tools that we present here, may be applied in stability theory of discrete dynamical systems. In the literature, there are conditions for $2-$dimensional systems [1], $3-$dimensional systems [3] and $4-$dimensional systems [4] involving the trace, the determinant and the sum of the minors of the Jacobian matrix. Beyond dimension 4, as far as I know, there are no studies for the linear stability conditions of discrete dynamical systems involving the trace, the determinant and the sum of the minors of the Jacobian matrix. The present work [5] solve this gap since these stability conditions may be determined from Jury's table using this new approach.

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Using non-smooth models to determine thresholds for microbial pest management

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Presentation type: Contributed Talk
The title of the special session: Population Dynamics and Related Topics

Releasing infectious pests could successfully control and eventually maintain the number of pests below a threshold level. To address this from a mathematical point of view, two non-smooth microbial pest-management models with threshold policy are proposed and investigated in the present paper. First, we establish an impulsive model with state-dependent control to describe the cultural control strategies, including releasing infectious pests and spraying chemical pesticide. We examine the existence and stability of an order-1 periodic solution, the existence of order-k periodic solutions and chaotic phenomena of this model by analyzing the properties of the Poincaré map. Secondly, we establish and analyze a Filippov model. By examining the sliding dynamics, we investigate the global stability of both the pseudo-equilibria and regular equilibria. The findings suggest that we can choose appropriate threshold levels and control intensity to maintain the number of pests at or below the economic threshold. The modelling and control outcomes presented here extend the results for the system with impulsive interventions at fixed moments.

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An Alternative Delayed Population Growth Difference Equation Model

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The title of the special session: Population Dynamics and Related Topics

We derive an alternative delayed population growth difference equation model based on a modification of the Beverton–Holt recurrence. By distinguishing the growth processes from the decline processes, we assume a delay only in the growth contribution, in the spirit of [1]. We additionally assume that those individuals that die during the delay, do not contribute to growth, an assumption introduced by Arino et al. in [2] and applied to the continuous logistic equation. The derived delay recurrence differs from the delayed logistic difference equation, known as the delayed Beverton–Holt or Pielou model, that was formulated as a discretization of the Hutchinson model. The analysis shows that if the time delay exceeds a critical threshold, the population goes extinct for all non-negative initial conditions. If the delay is below this threshold, the population survives and its size converges to a positive asymptotically stable equilibrium that is decreasing in size as the delay increases. These dynamics are arguably more realistic than the obtained dynamics for the (delayed) Pielou model, as one would expect that if a delay, i.e., time required before members of a population can contribute to growth, is too long, that the population would not be able to avoid extinction

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Population growth factor versus basic reproduction number in discrete-time structured two-sex population models

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Presentation type: Special Session Talk
The title of the special session: Population Dynamics and Related Topics

If $Q+H$ is the linearization of a nonlinear matrix model for a population with finite-dimensional structure at the zero vector (the extinction state) with nonnegative matrices Q and H and Q representing survival and individual development and H representing reproduction, $\mathbf{r}(Q+H)-1$ and $\mathbf{r}H(\mathbb{I}-Q)^{-1}-1$ have the same sign if $\mathbf{r}(Q)<1$ and a few reasonable extra conditions are satisfied [1]. Here $\mathbf{r}$ denotes the spectral radius. This result remains true if Q and H are positive linear bounded operators on an ordered Banach space X with generating normal cone X_+ [2]. If a two-sex population with mating is considered, H may only be a homogeneous continuous order-preserving operator on X_+.

We discuss to what degree the result remains valid in such a situation. The main difficulty is that it is not clear whether the spectral radius of $H \circ \sum_{n=0}^{\infty} \lambda^{-n-1} Q^n$ is a continuous function of $\lambda > \mathbf{r}(Q)$.

Applications to structured two-sex populations models will be presented.

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When can evolution destabilize system dynamics?: A case study for a predator-prey model

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The title of the special session: Population Dynamics and Related Topics

We consider a discrete-time predator-prey system in which the prey is evolving in response to an environmental stressor. Two types of evolutionary responses are compared: frequency-independent selection in which an individual's fitness is determined solely by the trait that it possesses, and frequency-dependent selection where an individual's fitness also depends on the traits possessed by other individuals. While it is shown that both types of evolution may destabilize the system dynamics, for the frequency-independent case this requires that evolution is sufficiently fast and results in a period doubling-route to chaos. In contrast, frequency-dependent selection may destabilize the system dynamics even for slow evolution via a Neimark-Sacker bifurcation. Moreover, unlike frequency-independent selection, we show that frequency-dependent selection may result in evolutionary suicide.

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Strong Allee Effect and Basins of Attraction in Discrete - Time Infectious Disease Models

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Presentation type: Special Session Talk
The title of the special session: Population Dynamics and Related Topics

Motivated by the Feline Immunodeficiency Virus, the virus that causes AIDS in cat populations, in this talk, we will use discrete-time infectious disease models with demographic strong Allee effect to examine the impact of the fatal susceptible - infected (SI) infections on two different types of density dependent growth functions: Holling type III or modified Beverton-Holt per-capita growth function (compensatory dynamics), and Ricker per-capita growth function with mating (overcompensatory dynamics). The occurrence of the strong Allee effect in the disease-free equation renders the SI population model bistable, where the two coexisting locally asymptotically stable equilibrium points are the origin (catastrophic extinction state) and either another fixed point or an intrinsically generated demographic period $k > 1$ population cycle. We will use the basic reproduction number, $R?0$, and the spectral radius, λ_k, to examine the structures of the coexisting attractors. In particular, we will show that the fatal disease is not only capable of enlarging or shrinking the basin of attraction of the catastrophic extinction state, but it can also fracture the basins of attraction into several disjoint sets. Thus, making it difficult to specify the asymptotic SI disease outcome in terms of all initial infections. The complexity of the basins of attractions appear to increase with an increase in the period of the demographic population cycles.

Invertible and Noninvertible Maps: Theory and Applications

Chairs: Laura Gardini and Iryna Sushko

List of Speakers

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Period 2 implies chaos: the hidden bridge between continuous and discontinuous worlds

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Presentation type: Special Session Talk
The title of the special session: Invertible and Noninvertible Maps: Theory and Applications

It is well known that bifurcation phenomena in discontinuous maps and continuous maps may be quite different. Recently, a novel approach for investigation of discontinuous maps has been suggested [1] which surprisingly combines several aspects of the dynamics commonly observed in both classes of maps. The key idea of this approach is to extend the definition of a discontinuous map in such a way that at the points of discontinuities, the function is considered to be *set-valued*, but, importantly, its solutions are analysed as single-valued trajectories *belonging* to a set (rather than set-valued themselves). By construction, in addition to all orbits existing in the original discontinuous map, such an extended map (referred to as a map with vertical branches) may have an infinite number of so-called *hidden orbits* including points inside the discontinuities.

Maps with vertical branches turn out to be useful for many purposes. In particular, they simplify the bifurcation analysis for discontinuous maps by describing the bifurcation structures in terms well-known for continuous maps. Here, hidden orbits provide "missing parts" of the bifurcation diagrams, turning border collision bifurcations at which a cycle appears "as if from nowhere" into the usual border collision flip and fold bifurcations [2]. Moreover, if a discontinuous map acts as a model of a system with a very fast but continuous switching process, a map with vertical branches helps to identify the dynamics which is neglected in such a modeling approach.

One of the most fundamental results proven for continuous 1D maps is the famous Sharkovsky theorem (which implies, in particular, the well-known rule “period three implies chaos”). It is also easy to demonstrate that this theorem does not apply to usual discontinuous maps, where, for example, a 3-cycle may exist alone, without any further coexisting cycles. However, it turns out that the Sharkovsky theorem can be proven not only for continuous but for maps with vertical branches as well [3]. The class of maps with vertical branches is broader than the class of continuous ones (every continuous function is connected but not vice versa). As the maps with vertical branches considered in our work are connected, the Sharkovsky theorem applies to them. Here, hidden cycles restore the Sharkovsky ordering, providing the cycles of all periods which are missing in the usual discontinuous map (without vertical branches).

A striking property of hidden orbits is that the existence of two distinct hidden cycles implies that a number of further hidden orbits exist as well, namely a countable number of other hidden cycles as well as an uncountable number of hidden aperiodic orbits. Although all these orbits may be located at a final number of points in the state space (the points of discontinuities and their preimages), their union can be seen as a hidden chaotic repeller. In the simplest case, the existence of a hidden fixed point and a hidden 2-cycle implies the existence of hidden cycles of all periods, which can be interpreted as an unexpected form of the well-known rule, namely “period two implies chaos”.

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Non connected basins in club goods adaptive binary games

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Presentation type: Special Session Talk
The title of the special session: Invertible and Noninvertible Maps: Theory and Applications

A dynamic adjustment mechanism, based on replicator dynamics (or imitate the better mechanism) in discrete time, is used to study the time evolution of a population of players facing a binary choice game with social influence, characterized by payoff curves that intersect at two interior points, also denoted as thresholds. The time evolution of the system is obtained by the repeated application of a noninvertible one-dimensional map. Such binary game can be interpreted as a club good game, in which players have to choose either joining or not the club in the presence of cost sharing, so that they can enjoy a good or a service provided that a "participation" threshold is reached. At the same time congestion occurs beyond a second higher threshold. These binary choice models, can be used (and indeed have been used in the literature) to represent several social and economic decisions, such as technology adoption, joining a commercial club, R&D investments, production delocalization, programs for environmental protection. Existence and stability of equilibrium points are studied, as well as the creation of more complex attractors (periodic or chaotic) related with overshooting effects. The study of some local and global dynamic properties of the evolutionary model proposed reveals that the presence of the "participation" threshold causes the creation of complex topological structures of the basins of coexisting attracting sets, so that a strong path dependence is observed. Moreover, if a memory effect is added, so that the new state depends not only on the previous state but on a combination of previous states as well, a two-dimensional noninvertible map is obtained. The dynamic effects of memory, both in the form of convex combination of a finite number of previous observation (moving average) and in the form of memory with increasing length and exponentially fading weights are investigated as well, and the creation of non connected or multiply connected basins of attraction is explained by using the method of critical curves.

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A stylized macro-model with interacting real, monetary and stock markets

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Presentation type: Special Session Talk
The title of the special session: Invertible and Noninvertible Maps: Theory and Applications

We consider an economy composed by linked and interacting real, monetary and financial sectors, and it is insipred by the model proposed in Blanchard (1981) and Semmler (2011). The setup for the real economy consists in a simple Keynesian good market in a closed economy where the production adjusts with respect to the aggregate demand. The money market is regulated by the standard assumption of an LM-equilibrium. In the financial market, the stock price is determined by a market maker who adjusts the price with respect to the current excess demand, which depends on the population of agents that can decide to participate or not in the financial market. Agents can decide to invest in the financial market or in the money market on the basis of an evolutionary selection mechanism regulated by the comparison between the return of the stock and the money market. Finally, we take into account both fundamentalist and chartist agents for the financial market. The link between the real and the financial sector is described by the dependence of private expenditure with respect to the stock price, resembling the fact that the status of the households and firms is positively/negatively affected by the good/bad performance of the financial market. The national income affects expected returns and, through the money market equilibrium, the interest rate, which both determine the eventual participation of agents in the financial market. Hence both real and money sectors affect the financial one. The influence of the financial sector on that monetary is through the national income, in which, as we said, private investments depend on the stock price. The link between real and monetary sectors is both direct, encompassed in the LM-equilibrium, and indirect through the financial sector. The model consists of a discrete time dynamical system that describes the interaction between the variables characterizing each market sector. We show the effect of the parameters characterizing the three sectors on the national income, the stock price and the share of agents that participate to the financial market at the equilibrium. Moreover, we investigate the possible emergence of out-of-equilibrium complex dynamics and the stabilizing/destabilizing role of each market sector. We show the emergence of both chaotic unpredictable fluctuations in the economic observables, as well as quasi-periodic dynamics resembling the businnes cycle.

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Bubbling, Riddling, Blowout and Critical Curves

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The title of the special session: Invertible and Noninvertible Maps: Theory and Applications

The method of critical curves can be exploited to study chaos synchronization phenomena in discrete dynamical systems with an invariant one-dimensional submanifold. Some examples of two-dimensional discrete dynamical systems, which exhibit synchronization of chaotic trajectories with the related phenomena of bubbling, on-off intermittency, blowout and riddles basins, are examined by the usual local analysis in terms of transverse Lyapunov exponents, whereas segments of critical curves are used to obtain the boundary of a two-dimensional compact trapping region containing the one-dimensional Milnor chaotic attractor on which synchronized dynamics occur. Thanks to the folding action of critical curves, the existence of such a compact region may strongly influence the effects of bubbling and blowout bifurcations, as it acts like a "trapping vessel" inside which bubbling and blowout phenomena are bounded by the global dynamical forces of the dynamical system.

Dynamics of the secant map near a critical 3-cycle

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Presentation type: Contributed Talk

We consider the secant method applied to a polynomial p as a dynamical system acting on the real plane. If the polynomial p has a local extrema at a point α then the secant map exhibits a 3-cycle at the point (α, α). We propose a simple model map T to explain the behavior of S^3 near the point (α, α). In many cases this 3-cycle has a non-empty basin of attraction and their boundary is related to the invariant manifold of some objects. This is a joint work with Ernest Fontich and Xavier Jarque.

Periodic solutions of perturbed globally periodic discrete dynamical systems

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The title of the special session: Invertible and Noninvertible Maps: Theory and Applications.

We consider non-autonomous N-periodic discrete dynamical systems of the form $x_{n+1} = F_n(x_n, \varepsilon)$, $x_n \in \mathbb{R}^m$, having when $\varepsilon = 0$, an open continuum of initial conditions such that the corresponding sequences are N-periodic. From the study of some variational equations of low order we obtain successive maps, that we call discrete Melnikov functions, such that the simple zeroes of the first one that is not identically zero control the initial conditions that persist as N-periodic sequences of the perturbed discrete dynamical system.

We apply these results to several examples. For instance, we prove that 1-dimensional N-periodic Abel difference equations can have at least $N-1$ isolated N-periodic solutions. This show that there is no upper bound for the number of isolated periodic orbits that Abel difference equations can have. This result is in contrast with what happens with linear or Riccati difference equations where these upper bounds exist and are 1, or 2, respectively. We also study periodic non-autonomous perturbations of some globally periodic difference equations like Lyness or Todd equations.

This talk is based on the papers [1, 2].

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Nonlinear dynamics in real economy and financial markets. The role of dividend policies in fluctuations

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This work investigates how dividend policies may influence the creation and propagation of cycles and chaotic behaviours between real economy and financial markets. We focus on the effect of a constant dividend policy on the stability of the aggregate economy, by means of a discrete dynamical framework in which managers, individuals and financial mediators coexist. We show the counter-intuitive effect of the dividend payout ratio: in a developed economy, an increase in dividends leads to a lower stock price level due to the cross feedback effect between markets. Moreover, in non-developed economies the choice of managers and individuals may not influence the propagation of fluctuations, while in developed economies, high payout ratios and high sensitivity to market trends trigger a cross feedback effect between the two markets that amplify their volatility and drags the whole economy into fluctuations and cycles.

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Topological properties of the immediate basins of attraction for the secant method

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The title of the special session: Invertible and Noninvertible Maps: Theory and Applications

We study the discrete dynamical system defined on a subset of R^2 given by the iterates of the secant method applied to a real polynomial p. Each simple real root α of p has associated its basin of attraction $\mathcal{A}(\alpha)$ formed by the set of points converging towards the fixed point (α, α) of S. We denote by $\mathcal{A}^*(\alpha)$ its immediate basin of attraction, that is, the connected component of $\mathcal{A}(\alpha)$ which contains (α, α). We focus on some topological properties of $\mathcal{A}^*(\alpha)$, when α is an internal real root of p. More precisely, we show the existence of a 4-cycle in $\partial\mathcal{A}^*(\alpha)$ and we give conditions on p to guarantee the simple connectivity of $\mathcal{A}^*(\alpha)$.

Dynamics of Certain Classes of Nonlinear Discontinuous Discrete Population Models

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Presentation type: Special Session Talk
The title of the special session: Noninvertible Maps: Theory and Applications

In this paper we present a survey of results about the dynamics of some disctere discontinuous population models. We study oscillations, the structure of semicycles, perodicity, invariant intervals, attractivity, and bifurcations. We focus on the classical Williamson's population model [4] and an equivalent model

$$x_{n+1} = (a - bh(x_n - c))x_n,$$

(h - Heaviside function, $x_0, c > 0$, $0 < b < 1 < a < b + 1$) which was used in modeling the spread of West Nile epidemic [2]. In addition, we consider properties of discontinuous Beverton-Holt type difference equation, whose carrying capacity and inherent growth rate both have jump discontinuities [3]. Also, we address the dynamics of a general nonlinear population model with two jump discontinuities exhibiting Allee-type effect [1]. Some preliminary results are introduced, few generalizations are presented, and several open problems are also discussed.

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Hybrid dynamical systems through noninvertible maps: some applications to economics

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The title of the special session: Invertible and Noninvertible Maps: Theory and Applications

We present some possible applications of hybrid models to economics. In particular, we consider evolutionary systems in which some state variables evolve in continuous time whereas the purely evolutionary dynamics evolve at discrete times. Through a discretization of the continuous variables, the problem is then reformulated by means of (mainly noninvertible) maps.

In the first example, the diffusion of *corporate social responsibility* is investigated by employing a hybrid evolutionary game where each firm chooses between being either socially responsible, which implies devoting a fraction of its profit to social projects, or non-socially responsible. Consumers prize socially responsible companies by paying a higher reservation price for their products. The hybrid evolutionary framework is characterized by quantity dynamics that describe the oligopolistic competition given firms' belief about the composition of the industry. At regular intervals of time, this belief is endogenously updated by a retrospective comparison on the profits obtained and on the basis of an evolutionary mechanism.

The second application proposes a bio-economic model of exploitation of renewable commercial resources. To take into account the typically continuous-time modeling of biological species and, instead, of the specialized harvesting activities, which by its nature cannot change continuously, the resulting dynamic system is again of the hybrid type, i.e. continuous for biological variables and discrete for the economic ones. We study the dynamic properties of the system through an equivalent three-dimensional noninvertible map to understand how economic parameters influence the long-run availability of resources.

On some pointwise periodic integrable piecewise linear maps

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We describe the dynamics of the piecewise linear maps

$$F_\alpha(x,y) = \begin{pmatrix} \cos(\alpha) & \sin(\alpha) \\ -\sin(\alpha) & \cos(\alpha) \end{pmatrix} \begin{pmatrix} x - \operatorname{sign}(y) \\ y \end{pmatrix} \text{ with } \alpha = \frac{\pi}{3}, \frac{\pi}{2} \text{ and } \frac{2\pi}{3}. \tag{1}$$

These maps correspond with the ones associated with the difference equations $x_{n+2} = -x_n - \rho x_{n+1} + \operatorname{sign}(x_{n+1})$ with $\rho \in \{-1, 0, 1\}$, studied by Chang, Wang and Cheng, see [1] and references therein.

The maps (1) are *pointwise periodic*, i.e. bijective in $\mathbb{R}^2$ and such that each point is periodic, but they are not *globally periodic*. For each of these maps we find a first integral. These first integrals exhibit unusual characteristics in the context of discrete dynamical systems: for instance, their set of values (the *energy levels*) are discrete, thus *quantized*. Furthermore, the level sets are bounded sets whose interior is like a necklace formed by a finite number of open tiles of a certain *regular or uniform tessellation*.

We detail the action of the maps on each invariant set of tiles in geometrical terms. Consider a map in (1) with first integral V, then: (a) We prove that F_α induces a dynamics between the M tiles of the necklace forming the level set $\{V(x,y) = c\}$, which *is conjugate to the one generated by an affine map* $h : \mathbb{Z}_M \to \mathbb{Z}_M$, which is k-periodic with $k \in \{M, M/2\} \cap \mathbb{N}$. Notice that, geometrically, F_α acts as a rotation among the tiles (beads) of the necklace. (b) We prove that *each tile is invariant by F^k, which is a rotation of order p around the center of the tile.* As a consequence, on each tile there is a k-periodic point (the center) and the rest of points are kp-periodic. The map h and all the values of M, k and p depend explicitly on the energy level c.

This is a joint work with Anna Cima, Armengol Gasull and Francesc Mañosas, see [2].

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Border collision bifurcations of chaotic attractors in 1D maps with multiple discontinuities

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Border collision bifurcations of fixed points and cycles are widely investigated in contrast to those of chaotic attractors, for which their transformations are usually associated with homoclinic bifurcations. For the most extensively studied class of piecewise smooth maps, i.e., 1D piecewise monotone maps with a *single* discontinuity, a chaotic attractor must include the border point, and thus, cannot collide with it [1]. If a map has *multiple* discontinuities, a direct border collision for a chaotic attractor becomes possible.

In the present paper we consider a 1D piecewise increasing, everywhere expanding linear map with two discontinuities and describe two types of bifurcations for chaotic attractors, which are not associated with any homoclinic bifurcation. In the simplest case, a chaotic attractor collides with a discontinuity point, which does not belong to this attractor, a phenomenon called an *exterior border collision bifurcation.* More sophisticated cases are grouped under the term *interior border collision bifurcation.* It occurs when some critical point (located inside a chaotic attractor) has exactly two preimages before the bifurcation. At the bifurcation moment this critical point coincides with another critical point, due to which one of its preimages disappears afterwards.

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Dynamics of a New Keynesian model with heterogeneous expectations: the role of monetary policy

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Modern monetary policy has emphasized that maintaining a stable monetary environment depends crucially on the ability of the policy regime to control inflation (and output) expectations. In fact, the activity of modern Central Banks is a form of management of expectations. The present work considers a standard New Keynesian model, described by a two-dimensional nonlinear map, to analyze the bifurcation structure when agents own heterogeneous expectations on inflation and output gap, and update their beliefs based on past performance. Agents are then allowed to switch among predictors over time. Depending on the degree of reactivity of the monetary policy to inflation and output deviations from the target equilibrium, different kind of dynamics may occur. Multiple equilibria and complicated dynamics, associated to codimension-2 bifurcations, may arise even if the monetary policy adheres to the Taylor principle. We show that if the monetary policy accommodates for a sufficient degree of output stabilization, complicated dynamics can be avoided and the number of coexisting equilibria reduces.
In the second part of the analysis, an arbitrarily large number of agents' beliefs is considered by applying the concept of Large Type Limit. In this respect, the intensity of choice or the spread of beliefs is crucial for the extent of the Central Bank to stabilize the economy. When the predictors are largely dispersed around the target, the Taylor principle is a requisite for stability; when the set of beliefs is somehow anchored to the target, stability can be achieved with a weaker monetary policy.

Transitions between several metastable consumption states in a stochastic consumer network

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We continue our study of behavioral change - as a transition between coexisting attractors - in the context of a *stochastic, non-linear consumption model* with interdependent agents. A *non-invertible* map constitutes the deterministic skeleton of the respective stochastic system. Our earlier work is extended by considering transitions potentially occurring between four coexisting deterministic attractors (4-cycle, 5-cycle, and 2 fixed points). While the immediate basins of the two fixed points possess a *smooth* boundary, the basin boundaries of the other attractors are of a *non-smooth* - possibly fractal - nature.

Relying on the indirect approach to the analysis of a stochastic dynamic system, we describe the potentially existing transition scenarios, and identify conditions in terms of behavioral and environmental parameters under which such transitions are likely to occur. Our stochastic analysis depends crucially on the stochastic sensitivity function technique due to [1] and its various spin-offs. Transitions occur as a consequence of specific relations prevailing between deterministic concepts (immediate basins of attraction) and stochastic concepts (confidence sets).

The key contribution of the paper consists in a solution to the problem of estimating noise levels for which transitions become likely (critical intensities) when the boundary of a basin is non-smooth. Secondly, we try to provide an economic interpretation of complex transitions between several coexisting consumption attractors.

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Heterogeneous agents with adaptively adjusted beliefs about prices in an evolutive Muthian cobweb model

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In the present work, starting from the 1D evolutionary Muthian cobweb model by Hommes and Wagener in [2], where the economy is populated by biased agents and unbiased fundamentalists, we investigate the effects generated by adaptively adjusted beliefs about prices of the good they produce. Like done in [2], we focus on the case in which the Muthian model is globally eductively stable in the sense of Guesnerie [1], being stable under naive expectations. In more detail, we first assume that just biased agents, being aware of the systematic errors they make in their forecasts, partially rely on an adaptive adjustment of beliefs, obtaining a 3D model, which has a unique steady state. Although in [2] the fundamental steady state is always stable and at most it can coexist with a period-two cycle, we prove that on increasing the reactivity of the evolutionary mechanism for the steady state in our setting there may be up to two stability thresholds and that the equilibrium can coexist with a quasiperiodic attractor. We contrast such results with those we find for the 4D setting, obtained by assuming that unbiased fundamentalists update their belief about the fundamental value partially relying on the same adaptive adjustment mechanism used by biased agents. Also in this case there exists just the fundamental equilibrium, whose stability conditions, under suitable homogeneity assumptions on the parameters, are stricter than those derived for the 3D framework, highlighting that the adoption of the considered adaptive mechanism by unbiased fundamentalists produces locally a destabilizing effect on the equilibrium, even if globally the complexity in the dynamics decreases with respect to the 3D setting. Under more general assumptions on the parameters related to biased agents and unbiased fundamentalists, we show that the 4D framework, still admitting the same unique steady state as in the homogeneous case, is able to generate quasiperiodic dynamics. We derive the local stability conditions for the equilibrium in the 4D setting under the more general parameter assumptions, too.

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Chaos, Border Collisions and Stylized Empirical Facts in an Asset Pricing Model with Heterogeneous Agents

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An asset pricing model with chartists, fundamentalists and trend followers is considered. A market maker adjusts the asset price in the direction of the excess demand at the end of each trading session. An exogenously given fundamental price discriminates between a bull market and a bear market. The buying and selling orders of traders change moving from a bull market to a bear market. Their asymmetric propensity to trade leads to a discontinuity in the model, with its deterministic skeleton given by a two-dimensional piecewise linear dynamical system in discrete time. Multiple attractors, such as a stable fixed point and one or more attracting cycles or cycles and chaotic attractors, appear through border-collision bifurcations. The multi-stability regions are underlined by means of two-dimensional bifurcation diagrams, where the border-collision-bifurcation curves are detected in analytic form at least for basic cycles with symbolic sequences LR^n and RL^n. A statistical analysis of the simulated time series of the asset returns, generated by perturbing the deterministic dynamics with a random-walk process, indicates that this is one of the simplest asset pricing models which are able to replicate stylized empirical facts, such as excess volatility, fat tails and volatility clustering.

The role of border collision bifurcations for emergence of business cycles

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As a starting point, we consider the well-known multiplier-accelerator model of business cycles introduced by Samuelson [1], which has been reconsidered by many authors and modified in various directions. The original Samuelson model, besides damped oscillations, also leads to divergence. To get sustained oscillations, we introduce two different types of governmental expenditures, and show that resulting two-dimensional continuous piecewise linear map is able to generate attracting cycles. The map is defined by three different linear functions in three different partitions of the phase plane, and this peculiarity influences the overall dynamics of the system. We show that similar to the classical Samuelson model, there is a unique feasible equilibrium as well as converging oscillations. However, for certain parameter values, this equilibrium undergoes a center bifurcation [2], and close to the related bifurcation value the attracting equilibrium coexists with attracting cycles of different periods. These cycles lose stability via a center bifurcation simultaneously with the equilibrium. Moreover, we show that attracting cycles of particular type also exist when the equilibrium becomes an unstable focus. For several families of attracting cycles, by introducing their symbolic representation, we obtain analytically the boundaries of the corresponding periodicity regions, associated with border collision bifurcations.

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Some Properties and Bifurcations of Two-dimensional Lotka-Voterra maps

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A particular system of two-dimensional Lotka-Volterra maps, $T_a : (x', y') = (x(a - x - y), xy)$, unfolding a map originally proposed by Sharkovsky for $a = 4$, is considered. We show the routes to chaos leading to the dynamics of map T_4. For map T_4 we show that even if the stable set of the origin O includes a set dense in an invariant area, the only homoclinic points of O belong to the $x-$axis, as well as the cycles leading to heteroclinic connections, while many internal cycles are snap-back repellers. We also show that a particular 6-cycle known analytically for map T_4 exists, and is known explicitly in closed form, for any $a \in (3, 4]$ appearing at a supercritical Neimark-Sacker bifurcation of the positive fixed point. Moreover, we show the existence of infinitely many $k-$cycles on the $x-$axis (for any $k > 3$), which are topological attractors of map T_a for $a \in (3.96, 4)$ and saddle cycles transversely attracting at $a = 4$.

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Uncertainty about fundamental, pessimistic and overconfident traders: a piecewise-linear maps approach

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We analyze a financial market model with heterogeneous interacting agents where fundamentalists and chartists are considered. We assume that fundamentalists are homogeneous in their trading strategy but heterogeneous in their belief about the asset's fundamental value. On the other hand, we consider that chartists, when they are optimistic become overconfident and they trade more than when they are pessimistic. Consequently, our model dynamics are driven by a one-dimensional piecewise-linear continuous map with three linear branches. We investigate the bifurcation structures in the map's parameter space and describe the endogenous fear and greed market dynamics from our asset-pricing model.

Global Dynamics Analysis of a Cournot Duopoly Game with R&D spillover and Heterogeneous Products

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Presentation type: Special Session Talk.
The title of the special session: Invertible and Noninvertible Maps: Theory and Applications.

In this paper, a dynamical Cournot model with R&D spillovers and heterogeneous products is established. The system is symmetric when the two firms are in same economic environment, and it is proved that both the diagonal and the coordinate axes are the one-dimensional invariant manifolds of the built system. The synchronization behaviors between the two firms are verified through calculating the transverse Lyapunov exponents. The impact of parameters, such as speed of adjustment and R&D efficiency, on the dynamical behaviors of the system is discussed. The topological structures of basins of attraction are analyzed through critical curves, and the formation mechanism of "holes" in the feasible region is numerically studied. In addition, some global bifurcation are also shown in this research.

This talk is based on papers [1,2].

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Doubling of a Closed Invariant Curve

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Presentation type: Special Session Talk
The title of the special session: Invertible and Noninvertible Maps: Theory and Applications.

In this work, we discuss how a closed invariant curve can undergo a doubling bifurcation. This question has already been addressed in [1] and [2, 3]. Gardini and Sushko [1], for instance, have proposed an explanation on how a closed invariant attracting curve, which appears in 3D smooth map via a Neimark-Sacker bifurcation can be transformed into a repelling one giving birth to a new attracting closed invariant curve which has doubled loops. Banerjee et al. [2] and Patra et al. [3] have described several scenarios for doubling of a closed invariant curve through local and global bifurcation in 3D piecewise smooth and piecewise linear maps.

We consider a hybrid (continuous-discrete) model comprising a continuous third-order plant with time delay under impulsive (or pulse-modulated) feedback. Such feedback constructs arise in modeling the pulsatile dynamics in endocrine regulation [4],[5]. In [4], the propagation of the continuous state vector through the firing instants of the impulsive feedback was shown to be governed by a piece-wise smooth discrete map with the dimension depending on the time-delay value. For small and intermediate time delays, the hybrid model has been previously demonstrated to exhibit a number of complex dynamic phenomena, including bistability, quasiperiodicity, and chaos [5]. In our study, we focus on an additional complexity that arises when the time delay is large (but not longer than three consecutive inter-impulse intervals) and the state space of the resulting map is five-dimensional. In this case, a doubling bifurcation of a closed invariant curve occurs.

The main steps of the scenario we report are as follows. First, a stable fixed point undergoes a Neimark-Sacker bifurcation leading to the appearance of a stable closed invariant curve. Thereafter, the unstable fixed point undergoes a flip bifurcation in the direction transverse to the 2D manifold including the fixed point and the stable invariant curve. This leads to the creation of another 2D invariant manifold which includes the stable closed invariant curve and the unstable 2-cycle. Topologically, this manifold represents a ball-shaped invariant set. Note that at this stage the stable closed invariant curve continues to exist. Eventually, this curve undergoes a supercritical doubling bifurcation causing it to change its stability and leading to a soft appearance of another closed invariant curve of a double-length around it.

In addition, we discuss a similar scenario leading to a subcritical variant of this bifurcation.

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Global Dynamics of Monotone Discrete Dynamical Systems

Chairs: Mustafa R.S. Kulenović

List of Speakers

Steve Baigent, Department of Mathematics, UCL, UK

E Cabral Balreira, Department of Mathematics, Trinity University, San Antonio, Texas, USA

Elliott Bertrand, Department of Mathematics, Sacred Heart University, Fairfield, CT, USA

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An introduction to the carrying simplex of competitive population models

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Presentation type: Special Session Talk
The title of the special session: Global Dynamics of Monotone Discrete Dynamical Systems.

The carrying simplex (CS) is an attracting invariant manifold of codimension one that is often found in models of competition in ecology. It is a higher dimensional analogue of the well-known carrying capacity. The CS was introduced for differential equation population models by Morris Hirsch [1] before later appearing in discrete-time populations [2, 3, 4, 5, 6, 7]. I will give a gentle introduction to the carrying simplex and its applications to population dynamics by combining theory, applications and some numerical simulations.

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Geometry and Global Stability of 2D Periodic Monotone Maps

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Presentation type: Special Session Talk
The title of the special session: Global Dynamics of Monotone Discrete Dynamical Systems.

We establish conditions to ensure global stability of a competitive periodic system from hypotheses on individual maps. From the conditions developed in [1] for maps defined on Euclidean spaces $\mathbb{R}^k_+$, of arbitrary dimension k, we now focus on planar competitive maps of Kolgomorov type in [2]. We show how conditions for global stability for individual maps will remain invariant under composition and hence establish a globally stable cycle. Our main theoretical contribution is to show that stability for monotone non-autonomous periodic maps can be reduced to a problem of global injectivity. We provide analytic conditions that can be checked and illustrate our results with important competition models such as the planar Leslie-Gower, Ricker, and Logistic maps.

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Dynamic Scenarios for Some Second-Order Monotone Difference Equations

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Presentation type: Special Session Talk
The title of the special session: Global Dynamics of Monotone Discrete Dynamical Systems

The first-order Beverton–Holt equation has been used as a classical model of population dynamics. A second-order generalization of this equation may take the following form:

$$x_{n+1} = \frac{af(x_n, x_{n-1})}{1 + f(x_n, x_{n-1})}, \quad n = 0, 1, \ldots. \tag{1}$$

Here f is a function that is nondecreasing in both variables, a is a positive constant, and both initial conditions x_0 and x_{-1} are nonnegative numbers in the domain of f. We will consider some global results for Eq. (1), particularly in the case when f is a linear or quadratic multivariate polynomial, as discussed in [1]. Particular attention will be given to the existence of period-two solutions. The monotonicity of Eq. (1) allows the theory of cooperative systems to be applied, and thus special cases of this equation will admit global dynamic scenarios presented in [2].

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Two Dimensional Continuous Lotka-Volterra Systems with Difference Equations

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Presentation type: Special Session Talk
The title of the special session: Global Dynamics of Monotone Discrete Dynamical Systems

We will consider the Discrete Lotka-Volterra Systems can be obtained either by different discretizations of continuous Lotka-Volterra systems or directly by modeling problems with difference equations.ÊWe willÊ come with the discretizations of two dimensional continuous Lotka-Volterra competitive and cooperative systems. We will also show the analysis of the global dynamics of these two models which include the local stability analysis, determining the global stable and unstable manifolds of all fixed points.

Preliminary Results on a Rational Equation with Quadratic Term

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Presentation type: Special Session Talk
The title of the special session: Global Dynamics of Monotone Discrete Dynamical Systems

We will present preliminary results about solutions of the following second-order rational difference equation with quadratic numerator and denominator:

$$x_{n+1} = \frac{\alpha + \delta x_n x_{n-1}}{B x_n + D x_n x_{n-1} + x_{n-1}},$$

where the coefficients are positive numbers, and the initial conditions x_{-1} and x_0 are nonnegative such that the denominator is nonzero.

Global Dynamics Results for a Class of Planar Cooperative Maps

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Presentation type: Special Session Talk
The title of the special session: Global Dynamics of Monotone Discrete Dynamical Systems

Sufficient conditions are given for planar cooperative maps to have the qualitative global dynamics determined solely on local stability information obtained from fixed and minimal period-two points. The results are given for a class of strongly cooperative planar maps of class C^1 on an order interval. The results can be easily extended to competitive maps. The maps are assumed to have a finite number of strongly ordered fixed points, and also the minimal period-two points are ordered in a sense. Some applications are included.

Stability analysis of a certain second order rational difference equation

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Presentation type: Special Session Talk
The title of the special session: Global Dynamics of Monotone Discrete Dynamical Systems.

In this paper we investigate the asymptotic stability of the following second order rational difference equation

$$x_{n+1} = C + A\frac{x_n^k}{x_{n-1}^p},\ n = 0, 1, \ldots$$

where A, C are positive real parameters. The initial conditions x_{-1}, x_0 are positive real numbers and $0 < k, p \in \mathbb{R}$. We discuss stability of the center manifold and 1-1 resonance case for the unique positive equilibrium. We also consider the case of the existence of Neimark-Sacker bifurcation and give the asymptotic approximation of the invariant curve near the unique equilibrium point.

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Dynamical Systems with Stable, Ergodic, and Historic Behavior

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Presentation type: Special Session Talk
The title of the special session: Global Dynamics of Monotone Discrete Dynamical Systems

The dynamical system theory studies the long term qualitative behavior of systems evolving in time. One of the important questions is that "*Will the system settle down to a steady state in the long run*?" If so, "*What is the possible steady state?*" If not, "*Do the time averages of the system exist and converge to some limit in the long run*?" The system is called *(asymptotically) stable* in the former case and *ergodic* in the latter case. To some extent, in these two cases, we can observe recurrent behavior of the dynamical system. However, there is the third possible scenario so-called *historic behavior* of the system which causes the non-existence of the time averages. The terminology *historic behavior* was coined by D. Ruelle [7] and the problem of describing the persistent family of dynamical systems with historic behavior was popularized by F. Takens [11, 12]. Recently [1, 2, 3, 4, 5, 6, 13], this problem was studied under the name of *Takens' Last Problem.* In this talk, we discuss one feature so-called *uniformly historic behavior* of a discrete dynamical system which reflects Ruelle's and Takens' argument for the non-existence of the time averages. Uniformly historic behavior will eventually cause the non-existence of multiply repeated time averages. As an application, we also provide a discrete-time Kolmogorov system of three-species predator-prey interactions in which uniformly historic behavior can be observed. This talk is based on the results of the papers [8, 9, 10].

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Global Behavior of Certain Higher Order Difference Equation

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Presentation type: Special Session Talk
The title of the special session: Global Dynamics of Monotone Discrete Dynamical Systems

We investigate the dynamical properties of the following higher order difference equation

$$x_{n+1} = A + B\frac{x_n}{x_{n-k}^r},$$

where parameters A, B and the initial values x_{-k},...,x_{-1} are arbitrary positive numbers, and $r > 0$ and $k \in \{1, 2, ...\}$ are fixed numbers. In some parametric space regions, we prove that the unique positive equilibrium point's local asymptotic stability implies global asymptotic stability.

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Topological and Low-Dimensional Dynamics

Chair: Lubomir Snoha

List of Speakers

Ana Anušić, University of São Paulo, Brazil

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Local structure of inverse limit spaces

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Presentation type: Special Session Talk
The title of the special session: Topological and low-dimensional dynamics

I will give an overview of what we know about the local structure of $\varprojlim(G, f)$, where G is a finite graph, and $f\colon G \to G$ is a continuous function. Such spaces can be realized as global attractors of homeomorphisms on $\mathbb{R}^3$ (or sometimes $\mathbb{R}^2$). The main goal is to characterize the existence, number, and type of points which are non-solenoidal, i.e. not locally homeomorphic to a zero dimensional set of arcs. For example, such points are often (but not always) limit points of the iterates of the critical set of f, while the recurrence of the critical set of f will often indicate the existence of endpoints. There are still a lot of open questions which I will discuss.

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On shifts and distributional chaos

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Presentation type: Special Session Talk
The title of the special session: Topological and low-dimensional dyanamics

Let (Σ, ρ) be the one-sided symbolic space of two symbols and σ the shift on it. We recall that the dynamical system (Σ, σ) is DC_1 or distributional chaotic in the sense of Schweizer-Smítal since it has an uncountable set S of pairs x, y that are of type DC_1. One question is if such S can be included in some of relevants sets in the space. In particular we prove that if R_σ and A_σ denotes the sets of *recurrent* and *almost periodic*, then $S \subset (R_\sigma \setminus A_\sigma)$

Using the notion of $DC_i - points$ for $i = 1, 2, 3$ related to the concentration of distributional chaos around some points from in any phase space, we prove a strongly situation in (Σ, ρ) in the sense that *every point* in such space is a $DC_1 - point$.

Such result is not true in the setting of general dynamical systems (X, f). It can be seen using Furstenberg families approach. There are examples of DC_2-spaces which are not DC_1.

There is also strong connection in the setting of general dynamical systems and *positive topological entropy*. A general result by Downarowicz states that positive topological entropy implies the space is DC_2. Such relevant result has been proved using properties on ergodicity of symbolic spaces.

We will provide some comments on the application of above problems in the setting of interval and triangular maps and recall some pending problems.

Metric Versus Topological Receptive Entropy of Semigroup Actions

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Presentation type: Special Session Talk
The title of the special session: Topological and Low-Dimensional Dynamics

We study the receptive metric entropy for semigroup actions on probability spaces, inspired by a similar notion of topological entropy introduced by Hofmann and Stoyanov (Adv Math 115:54–98, 1995). We analyze its basic properties and its relation with the classical metric entropy. In the case of semigroup actions on compact metric spaces we compare the receptive metric entropy with the receptive topological entropy looking for a Variational Principle. With this aim we propose several characterizations of the receptive topological entropy. Finally we introduce a receptive local metric entropy inspired by a notion by Bowen generalized in the classical setting of amenable group actions by Zheng and Chen, and we prove partial versions of the Brin–Katok Formula and the local Variational Principle. The talk is based on a joint paper [1] with Dikran Dikranjan, Anna Giordano Bruno, and Luchezar Stoyanov.

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The dynamics of a four-step feedback procedure to control chaos

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Presentation type: Special Session Talk
The title of the special session: Topological and Low-Dimensional Dynamics.

We make a description of the dynamics of a four-step procedure to control the dynamics of the logistic map introduced in [3]. Some massive calculations are made for computing the topological entropy with prescribed accuracy. This provides us the parameter regions where the model has a complicated dynamical behavior. Our computations also show the dynamic Parrondo's paradox "simple+simple=complex", which should be taking into account to avoid undesirable dynamics.

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Genericity of pseudo-arc in various classes of interval inverse limits

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Presentation type: Special Session Talk
The title of the special session: Topological and Low-Dimensional Dynamics

The pseudo-arc is besides the arc the only planar continuum (i.e. compact connected metric space) so that every of its proper subcontinua is homeomorphic to itself [1]. Its first description appeared in the literature about hundred years ago and due to many of its remarkable properties it is an object of much interest in several branches of mathematics. There are results indicating that pseudo-arc appears as a generic continuum in very general settings. For instance, Bing has proven that in any manifold M of dimension at least 2, the set of subcontinua homeomorphic to the pseudo-arc is a dense residual subset of the set of all subcontinua of M (equipped with the Vietoris topology). In this talk I will present a result which reveals that pseudo-arc is a generic object also in a certain measure theoretical setting; namely, I will show that the inverse limit of the generic Lebesgue measure preserving interval map is the pseudo-arc. I will also discuss several implications of this result.

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Periodic Points of Geometrically Integrable Maps in the Plane

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Presentation type: Special Session Talk
The title of the special session: Topological and Low-Dimensional Dynamics

The uniform approach to the concept of geometric integrability for discrete dynamical systems on invariant plane sets is suggested (see [2] - [3]). Geometric and analytic necessary and sufficient conditions for the geometric integrability of maps on invariant plane sets are proved [4]. Examples of geometrically integrable maps are given.

The solution of the coexistence problem of periodic points periods for these maps is given (see [4] - [5]). Obtained results are applied to description of the set of periodic points (least) periods of geometrically integrable maps with the quotient which is a symmetric Lorenz map. Here results of [1] are used.

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(Non-)Tameness among Toeplitz shifts

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Presentation type: Special Session Talk
The title of the special session: Topological and low-dimensional dynamics

Given a dynamical system (X, T)—where X is compact metric and T is a self-homeo on X—its *Ellis semigroup* is defined as the closure of the collection $\{T^n : n \in \mathbb{Z}\}$ in the space of self-maps on X. The Ellis semigroup is a cornerstone of the algebraic theory of topological dynamics. Unfortunately, quite often, it is quite nasty. This talk is about when the Ellis semigroup of Toeplitz shifts is well-behaved (or: *tame*).

Specifically, we aim at discussing a (very specialised) special case of the following theorem.

Let (X, T) be a Toeplitz shift of finite Toeplitz rank. Then (X, T) is tame if and only if its maximal equicontinuous factor has only countably many singular points.

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$\bar{f}$-continuity of entropy as a function of frequency-typical orbits

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Presentation type: Special Session Talk
The title of the special session: Topological and Low-Dimensional Dynamics

Assume that a sequence $x = x_0x_1x_2\dots$ is frequency-typical for a finite-valued stationary stochastic process $\mathbf{X}$. We prove that the function associating to x the entropy of $\mathbf{X}$ is uniformly continuous when one endows the set of all frequency-typical sequences with the f-bar pseudometric $\bar{f}$. As a consequence, we obtain the same result for the d-bar pseudometric $\bar{d}$. We also give an alternative proof of the Abramov formula for the Kolmogorov-Sinai entropy of the induced measure-preserving transformation.

This is joint work with Tomasz Downarowicz and Martha Łącka, see [1].

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Some advances in the study of ω-limit sets of Cournot maps

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Presentation type: Special Session Talk
The title of the special session: Topological and low-dimensional dyanamics

A main goal in discrete dynamics is to know the qualitative behaviour of the orbits generated by a map $F \in C(X, X)$, being X a topological space. For interval maps $f \in C(I, I), I = [0, 1]$, for each point $x \in I$, it is well-known the structure of its ω-limit sets, $\omega_f(x)$, the set of accumulation points of x under f. However, when we increase the dimension of the space, we only establish partial results about the ω-limit set (for instance, see [2] and [3] for triangular maps). In this talk, we focus on Cournot maps $F : I^2 \to I^2$, $F(x, y) = (f_2(y), f_1(x))$, with $f_1, f_2 \in C(I, I)$. These maps appear closely related to an economical process called the Cournot duopoly (see [5]). We will study the structure of ω-limit sets of Cournot maps having non-empty interior and we will provide some advances as well as some open problems in the case of empty interior. The results that we will present are collected in [1] and [4], where we have obtained a full description of ω-limit sets with non-empty interior.

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Borel complexity of sets of normal numbers via generic points in subshifts with specification

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Presentation type: Special Session Talk
The title of the special session: Topological and low-dimensional dynamics

We study the Borel complexity of sets of normal numbers in several numeration systems. Taking a dynamical point of view, we offer a unified treatment for continued fraction expansions and base r expansions, and their various generalisations: generalised Lüroth series expansions and β-expansions. In fact, we consider subshifts over a countable alphabet generated by all possible expansions of numbers in $[0, 1)$. Then normal numbers correspond to generic points of shift-invariant measures. It turns out that for these subshifts the set of generic points for a shift-invariant probability measure is precisely at the third level of the Borel hierarchy (it is a $\mathbf{\Pi}^0_3$-complete set, meaning that it is a countable intersection of F_σ-sets, but it is not possible to write it as a countable union of G_δ-sets). We also solve a problem of Sharkovsky–Sivak on the Borel complexity of the basin of statistical attraction. The crucial dynamical feature we need is a feeble form of specification. All expansions named above generate subshifts with this property. Hence the sets of normal numbers under consideration are $\mathbf{\Pi}^0_3$-complete.

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Special α-limit sets of Monotone maps on Regular Curves

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Presentation type: Special Session Talk
The title of the special session: Topological and Low-Dimensional Dynamics

In [1], Hero introduced, beside the usual limits sets ω-limit and α-limit, another kind of limit sets, called the special α-limit set to be as the union of the α-limit sets over all backward orbits starting at a point. It turns out for interval maps (resp. graph maps), many interesting properties have been established (see [1], [2], [4]) (resp. [3]). In this talk, we will present new results related to the structure of special α-limit sets and the continuity of the special α-limit maps of monotone maps defined on a large class of continua: the regular curves.

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Flexibility of entropies for piecewise expanding unimodal maps

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The title of the special session: Topological and Low-Dimensional Dynamics

We investigate flexibility of the entropies (topological and metric) for the class of piecewise expanding unimodal maps. We show that the only restrictions for the values of the topological and metric entropies in this class are that both are positive, the topological entropy is at most $\log 2$, and the metric entropy is not larger than the topological entropy.

In order to have better control on the metric entropy, we work mainly with topologically mixing piecewise expanding skew tent maps, for which there are only two different slopes. For those maps, there is an additional restriction that the topological entropy is larger than $\frac{\log 2}{2}$.

Moreover, we generalize and give a different interpretation of the Milnor-Thurston formula connecting the topological entropy and the kneading determinant for unimodal maps.

Mixing properties for expanding Lorenz maps on the interval

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Presentation type: Special Session Talk
The title of the special session: Topological and low-dimensional dynamics

Suppose that $f : [0,1] \to [0,2]$ is a continuous strictly increasing function which is differentiable on $(0,1) \setminus F$ where F is a finite set. Furthermore assume that $\beta := \inf f' := \inf_{x \in (0,1) \setminus F} f'(x) > 1$. Then there exists a unique $c \in (0,1)$ with $f(c) = 1$. Set $T_f x := f(x) - \lfloor f(x) \rfloor$, where $\lfloor y \rfloor$ is the largest integer smaller or equal to y. Such a map T_f is called an expanding Lorenz map. Note that T_f has a discontinuity at c.

For the case $\beta \geq \sqrt[3]{2}$ It topological transitivity and topological mixing of T_f is investigated. In the case $\beta \geq \sqrt[3]{2}$ and $f(0) \geq \frac{1}{\beta+1}$ the map T_f is topologically transitive. Furthermore it is also topologically mixing except in the case $f(x) = \sqrt[3]{2}x + \frac{2+\sqrt[3]{4}-2\sqrt[3]{2}}{2}$ for all $x \in [0,1]$.

Better results are obtained in the special case $f(x) = \beta x + \alpha$. Here one can completely describe the set of all (β, α) with $\sqrt[3]{2} \leq \beta \leq 2$ and $0 \leq \alpha \leq 2 - \beta$ such that T_f is topologically transitive. With three exceptions all of these topologically transitive maps are also topologically mixing.

According to Glendinning the map T_f is called locally eventually onto if for every nonempty open $U \subseteq [0,1]$ there are open intervals $U_1, U_2 \subseteq U$ and there are $n_1, n_2 \in \mathbb{N}$ such that ${T_f}^{n_1}$ maps U_1 homeomorphically to $(0,c)$ and ${T_f}^{n_2}$ maps U_2 homeomorphically to $(c,1)$. One calls T_f renormalizable if there are $0 \leq u_1 < c < u_2 \leq 1$ and $l, r \in \mathbb{N}$ with $l + r \geq 3$ such that ${T_f}^l$ is continuous on (u_1, c), ${T_f}^r$ is continuous on (c, u_2), $\lim_{x \to c^-} {T_f}^l x = u_2$ and $\lim_{x \to c^+} {T_f}^r x = u_1$. Then an example of a renormalizable and locally eventually onto expanding Lorenz map is given. Using a condition closely related to "locally eventually onto" it is shown that this condition is equivalent to T_f is not renormalizable.

Nonlinear Difference Equations and their Applications in Biological Dynamics

Chairs: Jianshe Yu, Jia Li and Bo Zheng

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Periodic motion for FPU lattice dynamical systems with the strongly indefinite case

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Presentation type: Special Session Talk
The title of the special session: Nonlinear Difference Equations and their Applications in Biological Dynamics.

In this talk, our main concern is how to obtain the existence of nontrivial solution for the general Fermi-Pasta-Ulam (FPU for short) type lattice dynamical system:

$$\ddot{q}_i = \Phi'_{i-1}(q_{i-1} - q_i) - \Phi'_i(q_i - q_{i+1}), \quad i \in \mathbb{Z},$$

where q_i denotes the co-ordinate of the i-th particle and Φ_i denotes the potential of the interaction between the i-th and the $(i+1)$-th particle. Our argument is variational. we obtain the ground state for FPU model with strongly indefinite case. Of particular interest is new and quite general approach: Non-Nehari method, which is developed recently . An interesting outcome from our result is that we can obtain the ground state solution without strict monotonous condition.

Traveling wavefronts of a delayed temporally discrete reaction-diffusion equation

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Presentation type: Special Session Talk
The title of the special session: Nonlinear Difference Equations and their Applications in Biological Dynamics.

With the growth of a single species with age structure on an unbounded domain as a prototype, we derive a delayed temporally discrete reaction-diffusion equation. The main result is on the existence of traveling wavefront solutions of the equation. We first transform the problem into that on the existence of fixed points of a mapping. Then by successfully constructing a pair of upper and lower solutions, we establish the existence of traveling wavefront by applying the upper-lower solution method.

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Mosquito control based on pesticides and *Wolbachia*

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Presentation type: Special Session Talk
The title of the special session: Nonlinear Difference Equations and their Applications in Biological Dynamics.

Mosquito-borne diseases have posed a serious thread to human heath around the world. Controlling vector mosquitoes is an effective method to prevent these diseases. In this paper, we incorporate consideration of releasing *Wolbachia*-infected mosquitoes and spraying pesticides to aim to reduce wild mosquito populations based on the population replacement model. We present the estimations for the number of wild mosquitoes or infection density in normal environment, and then discuss how to offset the effect of the heatwave, which can cause infected mosquitoes to lose *Wolbachia*-infection.

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Approximating gene transcription dynamics using steady-state formulas

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Presentation type: Special Session Talk
The title of the special session: Nonlinear Difference Equations and their Applications in Biological Dynamics

Understanding how genes in a single cell respond to dynamically changing signals has been a central question in stochastic gene transcription research. Recent studies have generated massive steady-state or snapshot mRNA distribution data of individual cells, and inferred a large spectrum of kinetic transcription parameters under varying conditions. However, there have been few algorithms to convert these static data into the temporal variation of kinetic rates. Real-time imaging has been developed to monitor stochastic transcription processes at the single-cell level, but the immense technicality has prevented its application to most endogenous loci in mammalian cells. In this article, we introduced a stochastic gene transcription model with variable kinetic rates induced by unstable cellular conditions. We approximated the transcription dynamics using easily obtained steady-state formulas in the model. We tested the approximation against experimental data in both prokaryotic and eukaryotic cells and further solidified the conditions that guarantee the robustness of the method. The method can be easily implemented to provide convenient tools for quantifying dynamic kinetics and mechanisms underlying the widespread static transcription data, and may shed a light on circumventing the limitation of current bursting data on transcriptional real-time imaging.

Discrete Dynamics of Dynamic Neural Fields

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Presentation type: Special Session Talk
The title of the special session: Nonlinear Difference Equations and their Applications in Biological Dynamics

Dynamics of Neural Fields are tools used in neurosciences to understand the activities generated by large ensembles of neurons. They are also used in networks analysis and neuroinformatics in particular to model a continuum of neural networks. They are mathematical models that describe the average behavior of these congregations of neurons, which are often in large amounts, even in small cortexes of the brain. Therefore, change of average activity (potential, connectivity,

ring rate, etc) are described using systems of partial different equations. In their continuous or discrete forms, these systems have a rich array of properties, among which the existence of nontrivial stationary solutions. In this talk, I will propose a discrete model for Dynamic Neural Fields based on nearly exact discretization schemes techniques. I will discuss the mathematical stability analysis of this model based on various types of kernels and corresponding parameters. Connection to graph theory will be shown. Simulations will be given for illustration.

Ground State Solutions of Discrete Asymptotically Autonomous Schrödinger Equations with Saturable Nonlinearities

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Presentation type: Special Session Talk.
The title of the special session: Nonlinear Difference Equations and their Applications in Biological Dynamics.

We consider the existence of ground state solutions for a class of discrete nonlinear Schrödinger equations with a sign-changing potential that converges at infinity and a nonlinear term being asymptotically linear at infinity. The resulting problem engages two major difficulties: one is that the associated functional is strongly indefinite and the other is that the classical methods such as periodic translation technique and compact inclusion method cannot be employed directly to deal with the lack of compactness of the Cerami sequence. New techniques are developed to overcome these two major difficulties. This enables us to establish the existence of a ground state solution and derive a necessary and sufficient condition for a special case. To the best of our knowledge, this is the first attempt in the literature on the existence of a ground state solution for the strongly indefinite problem under no periodicity condition on the bounded potential and the nonlinear term being asymptotically linear at infinity.

This is a joint work with Profs. Jianshe Yu and Zhan Zhou.

Impacts of dispersal on the dynamics of a delayed two-patch discrete SIR disease model

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Presentation type: Special Session Talk
The title of the special session: Nonlinear Difference Equations and their Applications in Biological Dynamics

In this paper, we propose a delayed discrete SIR disease model with saturate incidence rate and extend it to a patchy environment by taking the dispersal of susceptible individuals from one patch to the other into consideration. For the single-patch model, we establish the global threshold dynamics by the method of Lyapunov functionals. For the two-patch model, we show that the global dynamics of the disease-free equilibrium, two boundary endemic equilibria and the interior endemic equilibrium are determined by several threshold quantities. We also explore the impacts of the dispersal on the disease dynamics. Our interesting findings may provide some useful insights on how to properly manage the dispersal between different regions so that the disease is under control in involving regions.

Degenerate period adding bifurcation structure of a family of piecewise linear maps

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Presentation type: Special Session Talk
The title of the special session: Nonlinear Difference Equations and their Applications in Biological Dynamics

We will present a bifurcation structure for a family of 1D bimodal piecewise linear maps. This structure corresponds to border collision bifurcations affecting the outermost partitions of the state space. The case is rather degenerate compared to the general case usually addressed in the literature. The degeneracy affects both the type of border collision bifurcations and the number and location of the bifurcation points in the parameter space. We will present theoretical results yielding a complete description of both the border collision bifurcations and the bifurcation structure. We will show how these results allow us to extend partial results previously reported about a problem in the field of economics.

The study was motivated by a problem in the management of ecological populations. We will show how the results that we will present complete the description of the dynamics of the combined adaptive limiter control technique (CALC). We will provide numerical simulations showing potential risks and opportunities associated with the bifurcation structure from an ecological point of view. Moreover, we will provide examples of applications of our results to some well-known population models.

The talk is based on joint work with Daniel Franco and Frank Hilker [1, 2].

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Discrete dynamical models on *Wolbachia* infection frequency in mosquito populations with biased release ratios

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Presentation type: Special Session Talk
The title of the special session: Nonlinear Difference Equations and their Applications in Biological Dynamics.

In this paper, we develop two discrete models to study how supplemental releases affect the *Wolbachia* spread dynamics in cage mosquito populations. The first model focuses on the case when only infected males are supplementally released at each generation. This release strategy has been proved to be capable of speeding up the *Wolbachia* persistence by suppressing the compatible matings between uninfected individuals. The second model targets the case when only infected females are released at each generation. For both models, detailed model formulation, enumeration of the positive equilibria and their stability analysis are provided. Theoretical results show that the two models can generate bistable dynamics when there are three positive equilibrium points, semi-stable dynamics for the case of two positive equilibrium points. And when the positive equilibrium point is unique, it is globally asymptotically stable. Some numerical simulations are also offered to get helpful implications on the design of the release strategy.

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Propagation dynamics in a heterogeneous reaction-diffusion system under a shifting environment

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Presentation type: Special Session Talk
The title of the special session: Nonlinear Difference Equations and their Applications in Biological Dynamics

In this talk, we consider the propagation dynamics of a general heterogeneous reaction-diffusion system in a shifting environment. By developing the fixed-point theory for second order non-autonomous differential system and constructing appropriate upper and lower solutions, we show there exists a nondecreasing wave front with the speed consistent with the habitat shifting speed. We further show the uniqueness of forced waves by the sliding method and some analytical skills. In particular, we obtain the global stability of forced waves by applying the dynamical systems approach. Moreover, we establish the spreading speed of the system by appealing to the abstract theory of monotone semiflows. Applications and numerical simulations are also given to illustrate the analytical results.

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Periodic solutions with prescribed minimal period to Hamiltonian systems

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Presentation type: Special Session Talk
The title of the special session: Nonlinear Difference Equations and their Applications in Biological Dynamics

In this article, we study the existence of periodic solutions to second order Hamiltonian systems ($\ddot{x}+V'(x)=0, x\in\mathbb{R}^N$). Our goal is twofold. When the nonlinear term satisfies a strictly monotone condition, we show that, for any $T>0$, there exists a T-periodic solution with minimal period T. When the nonlinear term satisfies a non-decreasing condition, using a perturbation technique, we prove a similar result. In the latter case, the periodic solution corresponds to a critical point which minimizes the variational functional on the Nehari manifold which is not homeomorphic to the unit sphere.

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Global Dynamics for a Mosquito Population Suppression Model Interfered by *Wolbachia*-infected Males Causing Incomplete Cytoplasmic Incompatibility

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Presentation type: Special Session Talk
The title of the special session: Nonlinear Difference Equations and their Applications in Biological Dynamics.

In this talk, we will introduce a mosquito population suppression model, which includes the release of *Wolbachia*-infected males causing incomplete cytoplasmic incompatibility(CI). The model consists of two sub-equations switching each other, where the density-dependent birth rate of wild mosquitoes and incomplete CI effect are considered. Under the assumption that the waiting period T between two consecutive releases is greater than the sexual lifespan $\bar{T}$ of *Wolbachia*-infected males, we define a release amount threshold c^*, a CI intensity threshold s_h^* and a waiting period threshold T^* for the release. From a biological point of view, we assume $s_h > s_h^*$. When $g^* < c < c^*$, we prove that the origin E_0 is locally asymptotically stable if and only if $T < T^*$, and the model admits a unique globally asymptotically stable T-periodic solution if and only if $T \geq T^*$. When $c \geq c^*$, we show that the origin E_0 is globally asymptotically stable if and only if $T \leq T^*$, and the model has a unique globally asymptotically stable T-periodic solution if and only if $T > T^*$. Some numerical simulations are also provided to illustrate the theoretical results.

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A data-driven discrete model for the implementation of incompatible and sterile insect technique in Guangzhou

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Presentation type: Special Session Talk
The title of the special session: Nonlinear Difference Equations and their Applications in Biological Dynamics

SIT, the radiation-based sterile insect technique (SIT), has successfully suppressed field populations of several insect pest species, but its effect on mosquito vector control has been limited. IIT, the related incompatible insect technique (IIT), uses sterilization caused by the maternally inherited endosymbiotic bacteria *Wolbachia*——is a promising alternative way to suppress or even eradicate field mosquito populations. The implementation of IIT and SIT in Guangzhou enables near elimination of field populations of the world's most invasive mosquito species, *Aedes albopictus*. As collaborators, we take care of the mathematical modeling part in this project. In this talk, we will introduce the development of a discrete model for the implementation of IIT-SIT in Guangzhou started from 2015. This model is totally driven by the data from semi-cage experiments and has been included in [1]. Our mathematical model accurately described and predicted target population dynamics in the semi-field cage experiments, and supported the notion that 5:1 overflooding ratio of HC to wild-type males is sufficient for effective population suppression and/or elimination.

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Discrete Nonlinear Boundary Value Problems with the Mean Curvature Operator

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Presentation type: Special Session Talk
The title of the special session: Nonlinear Difference Equations and their Applications in Biological Dynamics

In this talk, I will introduce some results on the positive solutions for some nonlinear discrete Dirichlet boundary value problems with the mean curvature operator by using critical point theory. First, some sufficient conditions on the existence of infinitely many solutions are given. We show that, the suitable oscillating behavior of the nonlinear term near at the origin and at infinity will lead to the existence of a sequence of pairwise distinct nontrivial solutions. And by the strong maximum principle, we show that all these solutions are positive if the nonlinear term is nonnegative at zero. Then, the existence of at least two positive solutions is established when the nonlinear term is not oscillatory both at the origin and at infinity. Examples are also given to illustrate our main results at last.

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Stability and periodicity in a mosquito population suppression model being composed of two sub-models

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Presentation type: Special Session Talk
The title of the special session: Nonlinear Difference Equations and their Applications in Biological Dynamics.

In this paper, we propose a mosquito population suppression model which is composed of two sub-models switching each other. We assume that the releases of sterile mosquitoes are periodic and impulsive, only sexually active sterile mosquitoes can play a role in the mosquito population suppression process and the density-dependent survival probability is included in our model. For the release waiting period T and the release amount c, we find three thresholds denoted by T^*, g^* and c^* with $g^* < c^*$. We prove that the model generates a unique globally asymptotically stable T-periodic solution when either $c \in (g^*, c^*)$ and $T = T^*$, or $T > T^*$ for any $c > g^*$, and the origin is globally or locally asymptotically stable equilibrium when $c \geq c^*$ and $T \geq T^*$, or $c \in (g^*, c^*)$ and $T < T^*$, respectively. Finally, we give some numerical examples to illustrate our theoretical results and make some comparisons with some known results in the literature.

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Contributed Talks

Dynamical behavior of a P-dimensional system of nonlinear difference equations

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Presentation type: Contributed Talk.

In this work we study the periodicity, the boundedness of the solutions, and the global asymptotic stability of the positive equilibrium of the system of p nonlinear difference equations

$$x_{n+1}^{(1)} = A + \frac{x_{n-1}^{(1)}}{x_n^{(p)}}, \quad x_{n+1}^{(2)} = A + \frac{x_{n-1}^{(2)}}{x_n^{(p)}}, \ldots, \quad x_{n+1}^{(p-1)} = A + \frac{x_{n-1}^{(p-1)}}{x_n^{(p)}}, \quad x_{n+1}^{(p)} = A + \frac{x_{n-1}^{(p)}}{x_n^{(p-1)}}$$

where $n \in \mathbb{N}_0$, $p \geq 3$ is an integer, $A \in (0, +\infty)$ and the initial conditions $x_{-1}^{(j)}, x_0^{(j)}, j = 1, 2, \ldots, p$ are positive numbers.

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Existence and Uniqueness of Solutions to Discrete, Third-order Three-point Boundary Value Problems

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Presentation type: Contributed Talk

The purpose of this article is to move towards a more complete under- standing of the qualitative properties of solutions to discrete boundary value prob- lems. In particular, we introduce and develop sufficient conditions under which the existence of a unique solution for a third-order difference equation subjected to three-point boundary conditions is guaranteed. Our contributions are realized in the following ways. First, we construct the corresponding Green's function for the problem and formulate some new bounds on its summation. Second, we ap- ply these properties to the boundary value problem by drawing on Banach's fixed point theorem in conjunction with interesting metrics and appropriate inequalities. We discuss several examples to illustrate the nature of our advancements.

Synchronization in discrete-time dynamic systems with application in Biology

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Presentation type: Contributed Talk

Mathematical modeling of population dynamics has been attracted by many researchers over the last few decades. Specially, exponential difference equations have been used to model the interactions between different kind of population dynamics. Among these population models, Host-Parasitoid interactions play an important role in the ecosystem. One of the most important achievements in nonlinear and complex dynamics is the discovery of synchronized chaos. Synchronization happens when two events take place in synchrony at the same time and when time approaches infinity, the error between solutions of the first system and its synchronized one vanishes and approaches to zero. The synchronization between two dynamical systems is a well known phenomena occurring in Physics, Biology or Engineering and refers to a phenomenon that may occur when two or more oscillators are coupled. In this study, we develop a drive-response system by defining a convex continuous link function which maps the orbits of the drive system into the orbits of its coupled system and keeps the same qualitative dynamics. We represent an appropriate normal form for drive-response system and we obtain the conditions under which the solutions of drive and response system become completely synchronized. We provided a new concept in chaos synchronization, called, synchronization threshold, means that the solutions of drive and response system diverge from each other and lose the complete synchronization properties when they pass the threshold. We provide one application of this type of coupling to discover the synchronized cycles of generalized Nicholson-Bailey model (1)-(2). This model demonstrates a rich cascade of complex dynamics from stable fixed point to periodic orbits, quasi periodic orbits and chaos. Using the convex continuous link functions (5)-(6), we drive the response system (3)-(4) which inherits all the complex qualitative dynamics of GNB model (1)-(2) and mimics that certain properties of the motion which is shared between them.

$$x_1(n+1) = x_1(n)\, e^{r\left(1-\frac{x_1(n)}{k}\right)-r\, y_1(n)} \tag{1}$$

$$y_1(n+1) = x_1(n)\left(1-e^{-a\, y_1(n)}\right) \tag{2}$$

$$x_2(n+1) = p\, e^{r\left(1-\frac{p}{k}\right)-r\, q} \tag{3}$$

$$y_2(n+1) = p\,\left(1-e^{-a\, q}\right) \tag{4}$$

where

$$p = (1 - s)\, x_1(n) + s\, x_2(n) \tag{5}$$
$$q = (1 - s)\, y_1(n) + s\, y_2(n) \tag{6}$$

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On the Solution of a Third Order Linear Homogeneous Recurrence Relation with Variable Coefficients

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Presentation type: Contributed Talk

Recurrence sequences have been a central part of number theory for many years. Many number sequences are defined as linear recurrences, e.g. Fibonacci, Lucas, and Tribonacci numbers and their generalizations, Fibonacci-Narayana numbers, Pell-Padovan numbers [1, 2, 3, 4]. The linear recurrences have been extensively studied [5, 6] and solutions have been obtained basically using generating functions, shift operators, or matrix methods.

The aim of the talk is to briefly review previous results on the topic and present some new results on solving third order linear recurrence relations. We employ matrix methods that are useful in solving certain problems stemming from linear recurrence relations, and in obtaining some identities for special sequences. Using a matrix approach, we develop a new matrix method for solving linear recurrence relations and present explicit formulae for the general solution of third order linear homogeneous recurrence relations with variable coefficients, where the coefficient functions are assumed to be analytic. The solution seems to be more elegant and simple compared to other works, and involve Fibonacci numbers. We also obtain a summatory formula for the general solution of the recurrence relation in a special case. Some particular cases of the recurrence and examples with applications to combinatorics, especially to number sequences and polynomials, will be considered.

The method can be further generalized for higher order linear homogeneous recurrences with variable coefficients, discussion of which will conclude the talk.

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Nash–type equilibria for discretizations of nonlinear Dirichlet boundary value problems

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Presentation type: Contributed Talk

In this talk we consider system of nonlinear Dirichlet problems of the form

$$\begin{cases} -\ddot{x}_1(t) = f_1(t, x_1(t), \dots, x_m(t)) \text{ for } t \in (0,1), \\ \vdots \\ -\ddot{x}_m(t) = f_m(t, x_1(t), \dots, x_m(t)) \text{ for } t \in (0,1), \\ x_1(0) = \dots = x_m(0) = x_1(1) = \dots = x_m(1) = 0 \end{cases} \tag{1}$$

together with its discretizations

$$\begin{cases} -n^2\Delta^2 x_1[i-1] = f_1\left(\frac{i}{n}, x_1[i], \dots, x_m[i]\right) \text{ for } i = 1, \dots, n, \\ \vdots \\ -n^2\Delta^2 x_m[i-1] = f_m\left(\frac{i}{n}, x_1[i], \dots, x_m[i]\right) \text{ for } i = 1, \dots, n,, \\ x_1[0] = \dots = x_m[0] = x_1[n] = \dots = x_m[n] = 0 \end{cases} \tag{2}$$

where $\Delta^2 x[i] = x[i+2] - 2x[i+1] + x[i]$ and $x[i] = x\left(\frac{i}{n}\right)$. Following [1] and [2] we obtain existence and uniqueness of solutions to above problems together with their variational characterization in the form of a Nash–type equilibrium, see [3] for some other approach. We also discuss relations between solutions to (1) and (2) when $n \to \infty$ by showing that solutions to (2) converge to a unique solution to (1). The research will be bases on the theory of monotone operators supplied with the theory of M-matrices.

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Homogeneous Delayed Systems with Fixed Parameter

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Presentation type: Contributed Talk

The time delay fact is used in several research axes. Recently, it has been introduced in the many Mathematical field. In this work we resolve the stability of homogeneous delay systems based on the Lyapunov Razumikhin function in presence of a varying parameter. In addition, we show the stability of perturbed time delay systems when nominal part is homogeneous.

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Solving the linear difference equation with periodic coefficients via Linearly recurring sequences

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Presentation type: Contributed Talk

In this this talk, we give some explicit solutions of the homogeneous linear difference equations with periodic coefficients. For this purpose, we get around the problem by converting each equation of this class to an equivalent linear difference equation with constant coefficients. Second, we provide some expressions of the solutions via the combinatorial and the Binet formulas of linearly recurring sequences.

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Stability of a certain class of a host-parasitoid models with a spatial refuge effect

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Presentation type: Contributed Talk

A certain class of a host–parasitoid models, where some host are completely free from parasitism within a spatial refuge is studied. In this paper, we assume that a constant portion of host population may find a refuge and be safe from attack by parasitoids. We investigate the effect of the presence of refuge on the local stability and bifurcation of models. We give the reduction to the normal form and computation of the coefficients of the Neimark-Sacker bifurcation and the asymptotic approximation of the invariant curve. Then we apply theory to the three well-known host-parasitoid models, but now with refuge effect. In one of these models Chenciner bifurcation occurs. By using package Mathematica, we plot bifurcation diagrams, trajectories and the regions of stability and instability for each of these models.

A Stochastic Cellular Automaton with Avalanches, Distributions with Inverse-Power Asymptotics, and Motzkin Numbers Recurrence.

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Presentation type: Contributed Talk

A simple discrete toy model of seismicity in a form of 1-dimensional stochastic cellular automaton with avalanches, called Random Domino Automaton, is studied analytically. The defining rules of the automaton mimic general rules of a process of spatially separated accumulation of elastic energy coming from Earth's crust motions and its abrupt releases. Remarkably, the model proved to generate inverse-power-like distributions with exponential-like tails without addition of extra constraints, which is fitting well to the upper size limitation for maximal possible earthquake conditioned by mechanical strength of the crust. Thus, motivated by properties of earthquakes' statistics, we investigate details of generation of predefined inverse-power distributions with exponential tails.

The raison d'etre for the presentation is the immanent relationship between the model and the Motzkin numbers recurrence.

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On the existence and multiplicity of solutions for a class of Choquard logarithmic equations

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Presentation type: Contributed Talk

In this work we are concerned with the existence and multiplicity of non-trivial solutions for the following class of Choquard logarithmic equation

$$(-\Delta)_p^s u + |u|^{p-2}u + (\ln|\cdot| * |u|^p)|u|^{p-2}u = f(u) \quad \text{in} \quad \mathbb{R}^N,$$

where $N = sp$, $s \in (0,1)$, $p > 2$, $a > 0$, $\lambda > 0$ and $f : \mathbb{R} \to \mathbb{R}$ is a continuous nonlinearity with exponential critical growth. Based on variational techniques, considering f with critical growth, we obtain the the existence of non-trivial high level and ground state solutions. Then, considering f with subcritical growth, making use of genus theory, we prove the existence of infinitely many solutions.
Acknowledgements: The first author was supported by Coordination of Superior Level Staff Improvement-(CAPES) - Finance Code 001 and São Paulo Research Foundation-(FAPESP), grant ♯ 2019/22531-4, while the second author was supported by National Council for Scientific and Technological Development-(CNPq), grant ♯ 307061/2018-3 and FAPESP grant ♯ 2019/24901-3.

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About a System of Piecewise Linear Difference Equations with Many Periodic Solutions

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Presentation type: Contributed Talk

We consider the global behavior of the system of first order piecewise linear difference equations:

$$\begin{cases} x_{n+1} = |x_n| - y_n - b, \\ y_{n+1} = x_n - |y_n| - d, \end{cases} \quad n = 0, 1, 2, ..., (x_0, y_0) \in \mathbf{R}^2,$$

where the parameters b and d are any positive real numbers. We show that there exist an unstable equilibrium $(d; -b)$. We have a hypothesis that all solutions are evetually periodic solutions. It has been shown that there are no solutions with period 2, 3 and 4, but depending on the values of parameters b and d there are solutions with periods 5, 6, 7, 11, 12, 13, 16, 17, 18, 25. We have a hypothesis that there are no solutions with period 8, 9, 10, 14, 15. In general, depending on the values of parameters b and d there are solutions with many periods, but there also exist no solutions with many periods as well.

System with $b = 1$ and $d = 0$ at first was studied in [1]. Some new results about similar systems of first order piecewise linear difference equations can be found in [2] and [3].

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Difference equations for Restricted lattice path problems

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Presentation type: Contributed Talk

Lattice paths are counted by the nature of their step vectors that are confined to the positive octant. A path can go from any point to infinitely many others if no further restrictions apply, but each point on the path has only finitely many predecessors. However, to study lattice paths lying on or over a line having rational slope, linear difference equations with non-constant coefficients will be used to incorporate this restriction. We approach clear-cut solutions of such enumeration problems through generating functions, and we prove that the identity for generating functions is based on developing a specific method to compute the number of restricted lattice paths. We illustrate this method by counting some general lattice paths.

Let $x, m, \gamma \in \mathbb{Z}^N_{\geqslant}, P(\zeta) = \sum\limits_{0 \leqslant \gamma \leqslant m} c_\gamma \zeta^\gamma$ be a polynomial in $\zeta \in \mathbb{C}^N$. The inequality $0 \leqslant \gamma \leqslant m$ means that $0 \leqslant \gamma_j \leqslant m_j$ for all $j = 1, \ldots, N$. We denote $F_\gamma(\zeta) = \sum\limits_{x \geqslant \gamma} f(x)\zeta^x$ and $\Psi_\gamma(\zeta) = \sum\limits_{x \ngeqslant \gamma} \psi(x)\zeta^x$, where the inequality $x \ngeqslant \gamma$ means, that for at least one $j_0 = 1, \ldots, N$ the inequality $x_{j_0} < \gamma_{j_0}$ holds.

We first derive a general identity for the generating functions, We note that this theorem generalize the identity for generating functions given in [1].

Theorem. The generating function $F(\zeta) \in \mathbb{C}[[\zeta]]$ the identity is represented as

$$F(\zeta) = \frac{1}{P(\zeta)} \left(\sum_{0 \leqslant \gamma \leqslant m} c_\gamma \zeta^\gamma \Psi_{m-\gamma}(\zeta) + \sum_{x \geqslant m} P(\delta^{-I}) f(x) \zeta^x \right) \qquad (1)$$

holds, where $I = (1, \ldots, 1)$.

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Oscillation of Second Order Impulsive Neutral Difference Equations of Non-Canonical Type

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Presentation type: Contributed Talk

In this article, necessary and sufficient conditions for the oscillation of a class of non-linear second order neutral impulsive difference equations of the form:

$$\begin{cases} \Delta[a(n)\Delta(x(n)+p(n)x(n-\tau))]+q(n)F(x(n-\sigma))=0,\ n\neq m_j,\ j\in\mathbb{N} \\ \underline{\Delta}[a(m_j-1)\Delta(x(m_j-1)+p(m_j-1)x(m_j-\tau-1))]+r(m_j-1)F(x(m_j-\sigma-1))=0 \end{cases}$$

have been discussed for $p(n)\in(-1,0]$ with fixed moments of impulsive effect. Here, we assume that the nonlinear function is either strongly sublinear or strongly superliner. Some examples are given to illustrate our main results.
Keywords : Oscillation, nonoscillation, neutral difference equation, impulse, Lebesgue's dominated convergence theorem.

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Discrete Meijer G functions

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Presentation type: Contributed Talk

In recent years, a certain class of ${}_pF_q$ hypergeometric functions with variable parameters, called discrete hypergeometric functions, has been investigated. Numerous recent papers [1, 2, 3, 4, 5] investigate the properties of these functions and their associated special functions. In this talk, we will introduce a "discrete Meijer G" function by contour integration and discuss some of its properties, including how it can be thought of as a Meijer G function with variable parameters and its relation to the discrete hypergeometric series. This research was funded by the West Virginia NASA Space Grant Consortium.

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Linear Dynamics on L^p Spaces (Part I)

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Presentation type: Contributed Talk

In this first part, we start by recalling important concepts in Linear Dynamics such as transitivity, mixing, hiperciclicity, Li-Yorke chaos, hyperbolicity and shadowing. We provide an overview of these basic concepts and of some important tools, and of the main results in the literature concerning these fundamental topics in this research field, Linear Dynamics, which lies in between Dynamical Systems and Operator Theory, and has had a flurry of intriguing results, in particular, in the last 30 years. We introduce dissipative systems with the bounded distortion property, where our results for operators on L^p spaces are obtained.

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Backward asymptotics in S-unimodal maps

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Presentation type: Contributed Talk

While the forward trajectory of a point in a discrete dynamical system is always unique, in general a point has infinitely many backward trajectories. The union of the limit points of all backward trajectories through x was called by M. Hero [1] the "special α-limit" ($s\alpha$-limit for short) of x.

This concept plays a fundamental role in the construction of the graph of a dynamical system given by C. Conley [2], extending a seminal idea by S. Smale [3]: the nodes of the graph are the equivalence classes of all chain-recurrent points of the system and there is an edge from node A to node B if there is a point that asymptotes backward to A and forward to B. In a recent work with Jim Yorke [4], we studied the graph of the logistic map (more generally, of any S-unimodal map) and proved that there is a linear hierarchy between all nodes: nodes can be sorted as $N_0, N_1, \dots, N_p$, where N_p is the unique attractor and p is possibly infinite, so that arbitrarily close to each node N_i there are points that asymptote to N_j for each $j > i$. This behavior is not specific of S-unimodal mamps bvt appears also in higher-dimensional systems [5].

In this talk we show that, correspondingly to the hierarchy above, there is a hierarchy of $s\alpha$-limits of a S-unimodal map.

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Investigation of the Wave Solutions of the Nonlinear Partial Differential Equations Via Improved Bernoulli Sub-Equation Function Method

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In this study, traveling wave solutions are produced by using an improved Bernoulli sub- equation function method, considering a partial differential equation with strong non-linearity. The obtained solutions are examined with exact solution. 2D and 3D graphics of the obtained solution functions are drawn by determining the appropriate parameters. This technique appears as a suitable, applicable and efficient method to search for the exact solutions of nonlinear partial differential equations.

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Levels of local chaos for special Blocks Families

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Presentation type: Contributed Talk

In this work we going to study the properties of some chaotic properties via Furstenberg families, specially using other levels of Blocks families. Futhermore, We are going to relate the Li Yorke and Distribution chaos levels through the existence of certain categories of block families, which contain IP families and Weakly Thick families. Then, we are going to demonstrate that there is equivalence between chaotic localities in time actions that are closed under addition and multiplication, showing at the end some applications in types of shift systems from an ergodic point of view.

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L^p and almost sure convergence of an approximate method based on the Taylor expansion of the coefficients of Stochastic Functional Differential Equations

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Presentation type: Contributed Talk

A class of stochastic functional differential equations

$$dx(t) = a(x_t, t)dt + b(x_t, t)dw(t), \quad t \in [t_0, T], \tag{1}$$

with the given initial condition $x_{t_0} = \eta = \{\eta(\theta) \mid \theta \in [-\tau, 0]\}$ on a finite time interval is considered. In this paper, based on [1], the polynomial conditions are considered, instead of the usual assumptions that both the drift and diffusion coefficient satisfy the Lipschitz and linear growth conditions, as well as the assumption of the moment boundedness of the solution to the initial equation. An approximate equation is considered for any equidistant partition of the time interval. That equation has coefficients that are Taylor expansions of the coefficients of the initial equation. Taylor approximations require Fréchet derivatives since the coefficients of the initial equation are functionals. The solutions of thusly constructed equations converge in the L^p sense and almost surely towards the solution of Eq. (1) and the rate of the convergence is presented if those solutions satisfy some moment bounds. The rate of convergence increases if the orders of Taylor approximations for the drift and diffusion coefficient increase simultaneously. An example that illustrates the theoretical results and contains the proof of the existence, uniqueness and moment boundedness of the approximate solution is presented.

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q-regularly varying solutions of the half-linear q-difference equation

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Presentation type: Contributed Talk

The half-linear q-difference equation

$$D_q(p(t)\Phi(D_q(x(t)))) + r(t)\Phi(x(qt)) = 0, \ \ t \in q^{\mathbb{N}_0} = \{q^n : n \in \mathbb{N}_0\}, \ \ q > 1, \qquad (1)$$

where $\Phi(x) = |x|^\alpha \operatorname{sgn} x$, $\alpha > 0$, $p : q^{\mathbb{N}_0} \to (0, \infty)$, $r : q^{\mathbb{N}_0} \to \mathbb{R}$, will be analyzed in the framework of q-regular variation. The theory of q-Karamata functions will be used to establish necessary and sufficient conditions for the existence of q-regularly varying solutions under the assumption that the coefficient p of Equation (1) is q-regularly varying function and the coefficient r is an arbitrary function of eventually one sign. Moreover, in the case when r is eventually negative function, under the certain conditions, it will be examined whether all eventually positive solutions of Equation (1) are q-regularly varying. Furthermore, since Equation (1) can be transformed to the half-linear difference equation

$$\Delta(a(n)\Phi(\Delta(y(n)))) + b(n)\Phi(y(n+1)) = 0, \quad n \in \mathbb{N}_0,$$

where

$$a(n) = \frac{p(q^n)}{((q-1)q^n)^\alpha} \quad \text{and} \quad b(n) = (q-1)q^n r(q^n),\ n \in \mathbb{N}_0,$$

using generalized regularly varying sequences, obtained results are applied to the half-linear difference equation case.

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Linearization and Hölder Continuity for Nonautonomous Systems

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Presentation type: Contributed Talk

Let X and Y be two arbitrary Banach spaces. Moreover, let $(A_n)_{n\in\mathbb{Z}}$ be a sequence of bounded and invertible operators on X, $f_n\colon X\times Y\to X$, $n\in\mathbb{Z}$ a sequence of maps Lipschitz in the first variable and $g_n\colon Y\to Y$, $n\in\mathbb{Z}$ an arbitrary sequence of homeomorphisms. We consider the associated coupled system given by

$$x_{n+1}=A_nx_n+f_n(x_n,y_n),\quad y_{n+1}=g_n(y_n),\ n\in\mathbb{Z}. \tag{1}$$

In this talk, we will discuss sufficient conditions under which (1) is topologically equivalent to the uncoupled system

$$x_{n+1}=A_nx_n,\quad y_{n+1}=g_n(y_n),\ n\in\mathbb{Z}.$$

The talk will be based on the results obtained in [1].

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Antiprincipal solutions of dynamic symplectic systems on time scales

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Presentation type: Contributed Talk

In the presentation I will focuse on the a new concept of antiprincipal solutions at infinity for symplectic systems on time scales. This concept complements the earlier notion of principal solutions at infinity for these systems by Hilscher and and Šepitka (2016). In the article [1] created together with Hilscher we derive main properties of antiprincipal solutions at infinity, including their existence for all ranks in a given range and a construction from a certain minimal antiprincipal solution at infinity. We apply our new theory of antiprincipal solutions at infinity in the study of principal solutions, and in particular in the Reid construction of the minimal principal solution at infinity. In this work we do not assume any normality condition on the system, and we unify and extend to arbitrary time scales the theory of antiprincipal solutions at infinity of linear Hamiltonian differential systems and the theory of dominant solutions at infinity of symplectic difference systems.

Further I continue this investigation of the properties of recently introduced concept of antiprincipal solutions of symplectic system at infinity on time scales.

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Period-doubling and Neimark-Sacker bifurcations of a Beddington host-parasitoid model with a host refuge effect

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Presentation type: Contributed Talk

We explore the dynamics of a certain class of Beddington host-parasitoid models, where in each generation a constant portion of hosts are safe from attack by parasitoids, and the Ricker equation governs the host population. Using the intrinsic growth rate of the host population that is not safe from parasitoids as a bifurcation parameter, we prove that the system can either undergo a period-doubling or a Neimark–Sacker bifurcation when the unique interior steady state loses its stability. Then we apply the new theory to the following well-known cases: May's model, (S) model, Hassel and Varley (HV)-model, parasitoid-parasitoid (PP) model and (H) model. We use numerical simulations to confirm our theoretical results.

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Anisotropic Sobolev Embeddings and the Speed of Propagation for Parabolic Equations

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Presentation type: Contributed Talk

We consider a quasilinear parabolic Cauchy problem with spatial anisotropy of orthotropic type and study the spatial localization of solutions. Assuming that the initial datum is localized with respect to a coordinate having slow diffusion rate, we bound the corresponding directional velocity of the support along the flow. The expansion rate is shown to be optimal for large times.

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Robust stability of stochastic dynamical systems with varying delays

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Presentation type: Contributed Talk

This paper characterizes the robust second-moment stability of stochastic linear systems subject to varying delays. The delays assume a particular form suitable to represent packet loss in networked control systems, under the zero-order hold feedback. The proposed robust stability condition requires checking the spectral radius of an appropriate matrix that hinges upon a polytope. Due to this polytope's dependence, checking that spectral radius is difficult from the numerical viewpoint. As an attempt to solve the problem, we convert the polytope-based condition into a randomized approach. Namely, we present probability bounds that help us certificate the robust second-moment stability under high probability. A real-time electronic application illustrates the potential benefits of our approach.

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Study of bifurcation and chaos in discrete-time Rayleigh-Duffing oscillators

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Presentation type: Contributed Talk

In this work, by using the forward Euler's method we study the dynamics of discrete-time Rayleigh-Duffing oscillator. We apply the center manifold theorem to prove that the system has Hopf bifurcation and flip bifurcation. It is shown that the system undergoes chaotic behavior in the sense of Marotto's definition. Finally, we compute numerically the Lyapunov exponents to demonstrate sensitivity to initial conditions and chaotic behavior.

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Impulsive Difference Equations

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Contributed Talk

Impulsive difference equations are presented by showing the existence of periodic or bounded orbits, asymptotic behavior and chaos. Impulses are used to control the dynamics of the autonomous difference equations. Several examples are given including a model of supply and demand when Li-Yorke chaos is shown.

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Bifurcations and Applications of Multi-Parameter Continuous and Discrete Dynamical Systems

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Presentation type: Contributed Talk

We consider multi-parameter continuous and discrete dynamical systems and carry out their global bifurcation analysis [1]. First, using new bifurcational and topological methods, we solve *Hilbert's Sixteenth Problem* on the maximum number of limit cycles and their distribution for the 2D Holling-type quartic dynamical system [2], Leslie–Gower population dynamics system [3], Kukles cubic-linear system [4] and Euler–Lagrange–Liénard polynomial system [5]. Then, applying a similar approach, we study 3D polynomial systems and complete the strange attractor bifurcation scenario for Lorenz-type systems connecting globally the homoclinic, period-doubling, Andronov–Shilnikov, and period-halving bifurcations of their limit cycles which is related to *Smale's Fourteenth Problem* [6]. We discuss also how to apply our approach for studying global limit cycle bifurcations of multi-parameter discrete dynamical systems which model the population dynamics in biomedical and ecological systems.

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Critical point theorems in annular domains with applications to nonlinear equations

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Presentation type: Contributed Talk

Let H be a real, separable Hilbert space and let $J : Y \to \mathbb{R}$ be Gateaux differentiable such that $J^{'} : Y \to Y^{*}$ is demicontinuous and satisfies condition (S). Let $0 < r < R$. We consider nonlinear equations of the following type

$$J^{'}(u) = 0 \text{ for } u \in Y,\ r \leq \|u\| \leq R \tag{1}$$

under assumptions that J is bounded from below on $r \leq \|u\| \leq R$ and

$$J^{'}(u) + \lambda u \neq 0 \text{ for } \|u\| = r \text{ and all } \lambda > 0,$$

$$J^{'}(u) - \beta u \neq 0 \text{ for } \|u\| = R \text{ and all } \beta > 0.$$

Using the Ekeland Variational Principle and the Karush-Kuhn-Tucker Theorem we prove that there is some u_0 such that (1) holds. The applications to discrete and continuous problems are given.

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Cubic Splines on Time Scales

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In this talk we introduce the concept for cubic splines, cubic σ-splines, Hermite cubic splines and Hermite cubic σ-splines. They are deducted some of their properties. The talk is provided with applications of the defined splines on time scales in the theory of dynamic equations on time scales.

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Implicit linear difference equations over commutative rings

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Presentation type: Contributed Talk

Let R be a commutative ring with identity and $a, b, f_n \in R$ $(n \in \mathbb{N}_0)$. Consider the difference equation $bx_{n+1} = ax_n + f_n,\ n \in \mathbb{N}_0$ over R. If b is non-invertible, this equation is said to be implicit. Such equation may have no solutions over the ring R. For example, there is no sequence of integers such that satisfies the equation $3x_{n+1} = x_n + 1$. The following results describes classes of rings over that such an equation has a solution.

Theorem 1. Let R be a Noetherian local ring. Then for any $f \in R$ and any non-invertible $b \in R$ the equation $bx_{n+1} = x_n + f$ has a unique solution.

Theorem 2. Let R be a finite ring. Then for any $b \in R$ and $f_n \in R$ the equation $bx_{n+1} = x_n + f_n$ has a solution over R.

Over rings with a more complex structure we studied wider class of equations. Suppose R is a valuation ring of a complete field F with a non-Archimedean valuation $|\cdot|$.

Theorem 3. Suppose $a_j \in R$, $|a_j| < |a_0| = 1$ for $1 \le j \le m$, and $f_n \in R$. Then the equation $a_m x_{n+m} + a_{m-1}x_{n+m-1} + \ldots + a_0 x_n = f_n,\ n \in \mathbb{N}_0$ has a unique solution over R.

For the first and second order equation we can write down the explicit form of the solution. For instance, the solution of the equation $a_2x_{n+2} + a_1x_{n+1} + a_0x_n = f_n$ has the form of the series converges with respect to the non-Archimedean valuation:

$$x_n = \sum_{k=0}^{\infty} (-1)^{k+1} \sum_{j=0}^{[\frac{k}{2}]} (-1)^j \binom{k-j}{j} a_1^{k-2j} a_2^j a_0^{-k+j-1} f_{n+k}.$$

Specific examples of rings having the structure described above are the ring of p-adic integers and ring of formal power series. Thus this result is useful for finding a solution over the ring of integers and over the ring of polynomials.

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A Variational Framework for a Second Order Discrete Boundary Value Problem with Mixed Periodic Boundary Conditions

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Presentation type: Contributed Talk

Consider the second order difference equation

$$-\Delta(r(t-1)\Delta u(t-1)) = f(t, u(t)), \qquad t \in [1, N]_{\mathbb{Z}}, \tag{1}$$

together with the mixed periodic boundary conditions (BC)

$$u(0) = u(N), \quad r(0)\Delta u(0) = -r(N)\Delta u(N). \tag{2}$$

In order to study the existence of solutions, a variational framework consisting of an appropriate Banach space and an associated functional is constructed. This is needed to handle the asymmetry that occurs at the boundaries of the domain generated by the mixed periodic boundary conditions in (2). This new framework allows the study of the solutions of BVP (1), (2) defined in the standard way, i.e., defined on $[1, N]_Z$. The proofs make use of critical point theory. The approach used here may be combined with other results from critical point theory to lead to new results.

In addition to existence results, the problem of identifying a particular solution of the BVP that satisfies a pre-determined set of conditions is considered. It appears that this type of problem has not been considered for BVPs for either ordinary differential or difference equations.

Qualitative study of a p-dimensional system of difference equations

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Presentation type: Contributed Talk

The global asymptotic stability of the unique positive equilibrium point and the rate of convergence of positive solutions of the system of two recursive sequences has been studied recently. Here we generalize this study to the system of p recursive sequences $x_{n+1}^{(j)} = A + (x_{n-m}^{(j+1)mod(p)}/x_{n}^{(j+1)mod(p)})$, $n = 0, 1, \ldots, m, p \in \mathbb{N}$, where $A \in (0, +\infty)$, $x_{-i}^{(j)}$ are arbitrary positive numbers for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, p$. We also give some numerical examples to demonstrate the effectiveness of the results obtained.

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On generalized Archimedes-Borchardt algorithm

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Presentation type: Contributed Talk

This is a joint work with Justyna Jarczyk. We investigate convergence and invariance properties of the generalized Archimedes-Borchardt algorithm. The main tool is reducing the problem to an appropriate Gauss iteration process.

Critical linear difference equations

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Presentation type: Contributed Talk

Criticality was defined for second-order linear difference equations in [2] through specific positive solutions u^+ and u^- and was later extended by other authors for the even-order linear difference equations in [1]. There is a large space for further research in this field as there are not that many results about critical equations. Nevertheless, new results will soon appear in [4]. We show connections between criticality and given coefficients a_n, b_n of the studied equation

$$a_n y_{n-1} + b_n y_n + a_n y_{n+1} = 0$$

under different assumptions. Special attention is paid to the sequences c^+ and c^- which appear in the definition of the solutions u^+ and u^-. For example, we show that our equations are critical if and only if it holds $b_n = \frac{1}{c_n^-} + \frac{1}{c_n^+}$. Furthermore, we develop conditions under which we can asymptotically link sequences $c_n^\pm$ and coefficient a_n as $\lim_{n\to\infty} c^\pm = -\lim_{n\to\infty} \frac{1}{a_n}$, which is a natural generalization of a known result.

Other results are obtained for fourth-order equations, where we generalize the approach of articles [1, 3] to find that equation

$$-\triangle^4 y_n + \triangle\left(r_{n+1}\triangle y_{n+1}\right) = 0, \quad n \in \mathbb{Z},$$

is always 1-critical for a positive sequence r_n.

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Stability and Asymptotic Behaviour of a Linear Nabla Fractional Difference Equation

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Presentation Type: Contributed Talk.

Abstract: In this work, we discuss a few qualitative properties of the two-term linear nabla fractional difference equation

$$\big(\nabla^{\nu}_{\rho(a)}u\big)(t) = \lambda u(t-1), \quad t \in \mathbb{N}_{a+1},$$

where a, ν, $\lambda \in \mathbb{R}$, $0 < \nu < 1$, $\rho(a) = a - 1$, $\nabla^{\nu}_{\rho(a)}u$ denotes the ν-th Riemann–Liouville nabla fractional difference of u, and $\mathbb{N}_{a+1} = \{a+1, a+2, \cdots\}$. For this purpose, first we transform this nabla fractional difference equation into a Volterra difference equation of convolution-type. Using the well established qualitative theory of Volterra difference equations, we obtain sufficient conditions on boundedness, stability and asymptotic behaviour of solutions of the nabla fractional difference equation. Finally, we compare these properties with that of the two-term linear difference equation

$$\big(\nabla u\big)(t) = \lambda u(t-1), \quad t \in \mathbb{N}_{a+1}.$$

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Some Tools for Stability and Bifurcation for Discrete Dynamical Systems

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Presentation type: Contributed Talk

In this talk, I will present some interactive tools dealing with the stability for one- and two-dimensional discrete systems with the computer algebra system *SageMath*. These tools give information, geometrically, about the stability of the systems with parameters as well as the possible effects and types of bifurcations (if any) caused by the parameters. Using the tools, one can generate the basin of attraction of the fixed and periodic points, and also the stability regions in parameter space. This work is part of the book [4].

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Positive solutions of Thomas-Fermi type second-order difference equations with regularly varying coefficients

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Presentation type: Contributed Talk

Positive solutions of the nonlinear difference equation

$$\Delta(p_n|\Delta x_n|^{\alpha-1}\Delta x_n) = q_n|x_{n+1}|^{\beta-1}x_{n+1}, \quad n \geq 1, \quad \alpha > \beta > 0,$$

are studied under the assumption that p, q are regularly varying sequences. It is shown that with the help of discrete regular variation, complete information can be acquired about the existence of regularly varying solutions of this equation and their accurate asymptotic behavior at infinity.

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Novel delta and nabla Hardy-Copson type inequalities and their applications to dynamic equations

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Presentation type: Contributed Talk

Delta and nabla Hardy-Copson type dynamic inequalities are extended for the different exponents. The obtained inequalities are not only novel but also unify the continuous and discrete cases for which different exponents have not been considered so far. Moreover these inequalities are used to find necessary and sufficient conditions for the nonoscillation of the related half linear dynamic equations.

Diamond-alpha Hardy-Copson type dynamic inequalities and their extensions

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Presentation type: Contributed Talk

Although diamond alpha derivative and integral is constructed by convex linear combinations of delta and nabla ones, respectively, it is not straightforward to unify some well-known inequalities by diamond alpha calculus due to the lack of some important theorems. We establish diamond-alpha unification of delta and nabla Hardy-Copson type dynamic inequalities by developing a new method. Then we extend these inequalities for the different exponents. Moreover the obtained inequalities are not only novel but also unify the delta, nabla, continuous and discrete cases for which different exponents have not been considered so far.

A new splitting method for the Cox-Ingersoll-Ross process

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Presentation type: Contributed Talk

The Cox-Ingersoll-Ross (CIR) process is described by an Itô-type stochastic differential equation with a square-root diffusion and appears frequently in financial applications, for example in the pricing of interest rate derivatives. Solutions are almost surely (a.s.) non-negative; in fact under an additional parameter constraint called Feller's condition they are known to be a.s. positive. For Monte Carlo estimates, exact sampling from the conditional distribution is possible but computationally inefficient, and potentially restrictive if the innovating Brownian motion is correlated with that of another process.

A substantial literature has developed on the efficient numerical approximation of solutions. The challenge for numerical methods is to control error in spite of the unbounded gradient of the diffusion near zero, and to preserve the domain invariance of sampled trajectories. Indeed, Hefter and Jentzen [1], presented a bound on the strong convergence order of a class of uniform discretisations that includes those of Euler and Milstein type. This bound implies that CIR cannot be thus solved in a reasonable computational time if the intensity of the square-root diffusion is sufficiently dominant.

We propose a domain invariant numerical method for CIR based upon a suitable transform followed by a splitting. Moment bounds and strong L_2 and L_1 convergence rates of the scheme are available in a restricted parameter regime, and we compare the numerical performance to other recently developed methods. We can extend this method to all parameter values by introducing a non-uniform, adaptive mesh and "softening" the boundary at zero. Preliminary numerical evidence suggests that this modified approach yields strong convergence with nonzero rate, even for high intensity noise.

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The left-right nabla fractional difference equation with boundary-value condition

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Presentation type: Contributed Talk

In this paper, we introduce the following left-right nabla fractional discrete boundary-value problem

$$\begin{cases} {}_{T+1}\nabla_k^\alpha \left({}_k\nabla_0^\alpha(u(k))\right) + {}_k\nabla_0^\alpha \left({}_{T+1}\nabla_k^\alpha(u(k))\right) = \lambda f(k, u(k)), \quad k \in [1, T]_{\mathbb{N}_0}, \\ u(0) = u(T+1) = 0, \end{cases}$$

where $0 < \alpha < 1$ and ${}_0\nabla_k^\alpha$ is the left nabla discrete fractional difference and ${}_k\nabla_{T+1}^\alpha$ is the right nabla discrete fractional difference and $f : [1, T]_{\mathbb{N}_0} \times \mathbb{R} \to \mathbb{R}$ is a continuous function, $\lambda > 0$ is a parameter. We present the matrix structure form of this equation. Several examples are included to illustrate with $\alpha = 1/2$ and $\alpha = 3/4$.

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On the solution of a p-dimensional system of difference equations via Fibonacci numbers

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Presentation type: Contributed Talk

In this work we solve the following system of difference equations

$$x_{n+1}^{(j)} = \frac{F_{m+2} + F_{m+1} x_{n-k}^{((j+1) mod(p))}}{F_{m+3} + F_{m+2} x_{n-k}^{((j+1) mod(p))}}, \quad n, m, p, k \in N_0, j = \overline{1, p},$$

where $(F_n)_{n=0}^{+\infty}$ is the Fibonacci sequence. We give a representation of its general solution in terms of Fibonacci numbers and the initial values. Some theoretical justification related to the representation for the general solution are also given.

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Oscillation criteria for nonlinear difference equations with advanced type

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The purpose of this study is to analyze the first order nonlinear advanced difference equation

$$\nabla x(n) - p(n)f(x(\tau(n))) = 0, \ n = 0, 1, ...,$$

where $(p(n))$ is a sequence of positive real numbers and $(\tau(n))$ is not necessarily monotone argument. Also, some sufficient conditions for the oscillatory solutions of this equation are established. Finally, an example is given to demonstrate the results.

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Numerical Study on the Filtration Efficiency of Upper Generation of Human Lung

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Presentation type: Contributed Talk

In the present study we investigate the effect of flow of non-spherical nanoparticles through human airways and its adverse effect on the lower part of human lung generations under the periodic permeability [1, 2] of airways and oscillatory boundary conditions. An appropriate one-dimensional unsteady momentum equation in the cylindrical polar coordinate system is used by incorporating the idea of the shape factor of needle prolate nanoparticles. Filtration efficiency [3] of the lung from generations 5-16 is calculated using appropriate biofilter model. The effect of various physical parameters, such as mean permeability of media (K_0), the aspect ratio of particle (β), the orientation of particle with respect to the flow stream, Reynolds number (Re), and frequency of oscillation (f) are analyzed on the flow dynamics of air, particles and filtration efficiency of lung. Results show that the aspect ratio of a particle causes an increment in drag force and decrement in pressure gradient; and for parallel orientation velocity of particles increases than perpendicular orientation. Also, we found that the filtration efficiency of lung varies inversely with the value of mean permeability.

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Ostrowski-Grüss type inequality with application to the weight twopoint Radau integral formula

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Presentation type: Contributed Talk

Radau type integral formulae are numerical approximation formulae of semi-closed type and have many applications in matheatical analysis. Recently there have been proven some new results about error bounds of quadrature integral formulas motivated by famous Grüss and Ostrowski inequalities([1]). The aim of our paper is to extend these results and give some new error estimation to the weight case and to show applications to the weight Radau twopoint integral formulas ([2].

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Structural stability for dynamical systems on time scales

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Presentation type: Contributed talk

We consider a system on a time scale

$$x^{\Delta} = f(t,x), \quad x \in \mathbb{R}^n, \quad t \in \mathbb{T} \tag{1}$$

where the time scale $\mathbb{T}$ is an unbounded closed subset of $\mathbb{R}$.

Definition. We say that the system (1) is *structurally stable* if for any $\varepsilon > 0$ there exists a $\delta > 0$ such that for any $g(t,x): \quad |g(t,x)| < \delta, \quad |g'_x(t,x)| < \delta$ and any $t_0 \in \mathbb{T}$ there is a homeomorphism h of the space $\mathbb{R}^n$ such that

$$|\varphi_f(t,x_0) - \varphi_{f+g}(t,h(x_0))| < \varepsilon$$

for any $x_0 \in \mathbb{R}^n$, $t \in \mathbb{T}$. Here $\varphi_f(t,x_0)$ and $\varphi_{f+g}(x_0)$ are solutions of systems (1) and

$$x^{\Delta} = f(t,x) + g(t,x) \tag{2}$$

with initial conditions $x(t_0) = x_0$.

For systems of ordinary differential equations, conditions for global structural stability were obtained in [1], see also [2]. It was proved that a system is structurally stable if its linearizations are uniformly hyperbolic on families of segments.

We formulate and prove an analog of this statement for time scale systems. Although the result is very similar to that for ordinary differential equations, the proof for the time scale case is significantly different. We need to use specific approaches of time scale systems theory [3]. Remarkably, the classical results for structural stability of autonomous systems of ODEs, obtained by C.Robinson [4], are, in general, non-applicable for systems on time scales (even for the autonomous ones).

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On continuity of fractional iterates of Brouwer homeomorphisms

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Presentation type: Contributed Talk

We describe a method of construction of continuous orientation preserving iterative roots of a Brouwer homeomorphism for which there exists a family of pairwise disjoint invariant lines covering the plane. To obtain such roots we use a matching property for invariant lines contained in the boundaries of maximal parallelizable regions of the considered Brouwer homeomorphism. This property allows an iterative root defined on a parallelizable region to be extended on the boundary of this region which is a key step in the presented construction.

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Linear Dynamics on L^p Spaces (Part II)

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In this second part, we present some of our recent results. In particular, we focus on the relations between generalized hyperbolicity and shadowing. As it is well-known, hyperbolicity implies shadowing but the equivalence is not always true. Therefore, the notion of generalized hyperbolicity is an important bridge between hyperbolicity and the shadowing. We characterize the shadowing property for composition operators on dissipative systems with the bounded distortion property, and in particular, we show that, in this contest, the shadowing property and the generalized hyperbolicity coincide. We provide tools for the construction of examples, using distributions.

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Vaccination against morbidity risks: an evolutionary dynamics analysis

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Presentation type: Contributed Talk

For diseases in which vaccination is not compulsory, individuals take into account different aspects when deciding between to vaccinate or not. Namely, the decision depends on the morbidity risks from both vaccination and infection, and also depends on the probability of being infected, which varies with the course of the disease and the decisions of all other individuals. In this work, we study the evolution of the vaccination strategies depending upon the morbidity risks and upon the parameters of the epidemic model. We give a special emphasis to the possibility of reinfection. In [1], Martins and Pinto introduced the evolutionary vaccination dynamics for a homogeneous vaccination strategy of the population, where the individuals change their strategies over time, such that their payoffs increase. Here, we will also consider the dynamical evolution of the perceived morbidity risks and we analyze the changes provoked on the population vaccination strategy.

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Learning to Play Nash Equilibrium in Chaotic Dynamics

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Presentation type: Contributed Talk

In a bounded rational game where players cannot be as super-rational as in Kalai and Leher (1993), are there simple adaptive heuristics or rules that can be used to secure convergence to Nash equilibria? Robinson (1951) showed that for certain types of games, such rules exist. Nevertheless, the types of games to which they apply are pretty restrictive. Following Hart and Mas-Colell (2003) terminology, are there games with uncoupled deterministic dynamics in discrete time that converge to Nash equilibrium or not? Young (2009) argues that if an adaptive learning rule follows three conditions – (i) it is uncoupled, (ii) each player's choice of action depends solely on the frequency distribution of past play, and (iii) each player's choice of action, conditional on the state, is deterministic – no such rule leads the players' behavior to converge to the Nash equilibrium. This paper shows that there are simple adaptive rules that secure convergence, in fact, fast convergence, in a fully deterministic and uncoupled game. We use the Cournot model with nonlinear costs and incomplete information for this purpose and illustrate that this convergence can be achieved without any coordination of the players' actions.

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Occasionally Binding Constraints in the New Keynesian Model

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Presentation type: Contributed Talk

Our main goal is to analyze a standard macroeconomic model with occasionally binding constraints (OBCs) in this paper. This type of problem is quite ubiquitous in macroeconomics but is usually ignored for high computational demands. For example, we can find OBCs in models with a zero lower bound constraint on interest rates, in models with occasionally binding collateral constraints, downward nominal wage rigidities, irreversible investment, irreversible natural resources, or discrete decision making in Markov decision processes.

In particular, we will focus on discussing the New Keynesian Model with a typical Taylor Rule on the nominal short-term interest rate. In this framework, we can consider two different states (a "normal" state and the "Zero Lower Bound" on interest rates), modeled according to a Markov process. The solution is obtained by the method of time iteration and follows the approaches recently presented by Sunakawa and Hirose (2019) and Rendahl (2017).

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A new method using Bell polynomial for solution of some nonlinear functional differential equations

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Presentation type: Contributed Talk

In the present study, a new method is proposed to obtain a solution of nonlinear delay equations (NDDDEs) with initial and boundary conditions. The presented method is a combination of differential transform and partial ordinary Bell polynomials. Differential transform is based on Taylor series and to deal with nonlinearity Bell polynomials are applied. The applications of the method have been illustrated through some test problems. Convergence results are presented. The error estimate is discussed too. The presented method is not only reliable in obtaining the solution of such problems in series form with high accuracy, but it also guarantees considerable saving of calculation volume and time as compared to other methods. The numerical calculations are performed using Mathematica software version 11.

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A new parameter-uniform discretization of semilinear singularly perturbed problems

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Presentation type: Contributed Talk

In the framework of finite difference methods, semilinear singularly perturbed problems have been widely solved on non-uniform grids. In this paper, we propose a fitted operator finite difference method which uses uniform grids. We first use the quasilinearization technique to transform the semilinear singularly perturbed equation into a system of linear equations. The system is then solved via a fitted operator method. We show that error estimates for the proposed method are independent of the perturbation parameter. Also, we carry out numerical illustrations to confirm the robustness of the method.

Fuzzy molecular modeling of nabla fractional difference complex-valued molecular models of mRNA and protein in regulatory mechanisms

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Presentation type: Contributed Talk

This paper addresses the problem of leakage effects on Mittag-Leffler synchronization of T-S fuzzy fractional-order discrete-time complex-valued molecular models of mRNA and protein in regulatory mechanisms with two kinds of regulation functions, respectively. A novel approach is proposed to effectively deal with the joint effects from leakage delay and time varying delay for the class of T-S fuzzy fractional-order discrete-time complex-valued genetic regulatory networks (FDTCVGRNs) under consideration. By employing Lyapunov function method and Caputo fractional difference inequalities, several effective conditions according to algebraic inequality and complex-valued linear matrix inequalities (LMIs) are deduced to guarantee the Mittag-Leffler synchronization of the addressed FDTCVGRNs. Moreover, Mittag-Leffler synchronization problem of the nonlinear regulation function in complex-valued molecular models to study on the basis of genernal regulation function and linear threshold one. Compared with existing results in the literature, we also show that our results are less conservative than existing ones with these illustrative FDTCVGRNs. Finally, the simulation results of two numerical example are demonstrated, which explains the validity of the proposed method.

Periodicity on max-type difference equations

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Presentation type: Contributed Talk

In the last decades the interest on the behaviour of the solutions of max-type difference equations has rapidly grown. In this sense, we can find a wide range of work in the literature dealing with the periodic character of its solutions, boundedness, convergence, stability... (for instance, see the monograph [4]).

In the present talk, we consider the max-type equation

$$x_{n+4} = \max\{x_{n+3}, x_{n+2}, x_{n+1}, 0\} - x_n,$$

with arbitrary real initial conditions. We describe completely its set of periods $\mathrm{Per}(F_4)$, as well as its associate periodic orbits. Moreover, we prove that there exists a natural number $N \notin \mathrm{Per}(F_4)$ for which

$$\{N + m : m \geq 1, m \in \mathbb{N}\} \subset \mathrm{Per}(F_4).$$

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The solvability of discrete boundary value problems on the half-line

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Presentation type: Contributed Talk

This talk is devoted to the study of the existence of solutions to second-order nonlinear boundary value problem on the half-line of the form

$$\begin{cases} \Delta\left(a(n)\Delta x(n)\right) = f(n, x(n), \Delta x(n)), & n \in \mathbb{N} \cup \{0\}, \\ \alpha x(0) + \beta a(0)\Delta x(0) = 0, & x(\infty) = d, \end{cases}$$

where $d, \alpha, \beta \in \mathbb{R}$, $\alpha^2 + \beta^2 > 0$. To achieve our goal, we use Schauder's fixed point theorem and the perturbation technique for Fredholm operator of index zero. The necessary condition for the existence of solutions to the considered problem are presented, too.

Control of multi-agent dynamical systems: the coverage problem

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Presentation type: Contributed Talk

The talk considers the system composed of finite number of homogeneous dynamical systems (which can be interpreted as individuals or autonomous systems), so-called agents, which are (locally) communicating, interacting and cooperating with each other. For such multi-agent systems, the control strategies can target different objectives according to the level of cooperativeness. We investigate the problem of optimal coverage of a constrained predetermined target area, ideally leading to a static configuration. This problem is also known as the deployment problem and it is shown that a tracking strategy concentrating on the Chebyshev center of Voronoi cell surrounding each agent can lead to a robust behaviour. However, the fact that the Voronoi partition is dynamically updated leads to complex behaviours involving potential switching. The problems is shown to be equivalent to the stability analysis for a system governed by a time-varying linear difference equations. Stability criteria are described in this context base on a parameter dependent Lyapunov function. Furthermore, we consider the configuration of control system affected by transmission delay d. Delays are phenomena that cause a time-shift in the input signal which can be translated into delay difference equations for the closed-loop system. It is shown that such multi-agent systems affected by delays present complex behaviours, which deserve particular attention for a complete description in terms of clustering, limit behaviour and convergence.

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Another characterization of hyperbolic diameter diminishing to zero IFSs

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Presentation type: Contributed Talk

In this paper we provide another characterization of hyperbolic diameter diminishing to zero iterated function systems that were studied in [R. Miculescu, A. Mihail, Diameter diminishing to zero IFSs, arXiv:2101.12705]. The primary tool that we use is an operator $H_{\mathcal{S}}$, associated to the iterated function system $\mathcal{S}$, which is inspired by the similar one utilized in Mihail (Fixed Point Theory Appl 2015:15, 2015). Some fixed point results are also obtained as byproducts of our main result.

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The Modified Formal Variational Structure and Variational Integrator

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Presentation type: Contributed Talk

The formal Lagrangian method and self-adjointness technique have been found useful for computing conservation laws of non-variational differential, semi-discrete or finite difference equations [2, 4, 5]. Although it was shown that every system of differential equations can be embedded into a bigger variational system [1], how to recover the original system through a proper reduction has attracted a lot of attention, particularly for general differential equations. In this talk, we will give an introduction to the modified formal variational structure that is, in principle, applicable to all differential equations [3, 6]. We will also show its potential applications for deriving variational integrators. Worked examples will be provided.

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Linearization of hyperbolic logarithmic transseries and Dulac germs

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Presentation type: Contributed Talk

Logarithmic transseries are formal sums of powers and iterated logarithms with real coefficients. We consider hyperbolic logarithmic transseries $f = \lambda z + ...$, $0 < \lambda < 1$. In dynamics, transseries are associated with the Dulac's problem of non-accumulation of limit cycles on a hyperbolic or semi-hyperbolic polycycle of an analytic planar vector field, which is solved independently by Ilyashenko and Écalle. Every real analytic germ on $\langle 0, d\rangle$, $d > 0$, with Dulac series as its asymptotic expansion, which can be expanded on some complex domain called standard quadratic domain, is called Dulac germ. We obtain normal forms of hyperbolic transseries, which are, roughly speaking, *the simplest* transseries which are conjugated to the original one. In fact, we generalize results from [1], but using different techniques. In particular, we obtain normalizations using Banach fixed point theorem. By Koenigs' theorem, we know that complex analytic diffeomorphism $f(z) = \lambda z + o(z)$, $0 < |\lambda| < 1$, can be linearized. We find necessary and sufficient condition for hyperbolic transseries to be linearized and we apply these results to prove linearization theorem for hyperbolic Dulac germs on standard quadratic domains, which can be seen as a generalization of the mentioned Koenigs' theorem.

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On the solvability of the discrete $s(x, \cdot)$-Laplacian problems on simple, connected, undirected, weighted and finite graphs

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Presentation type: Contributed Talk

The solvability of the discrete $s(x, \cdot)$-Laplacian problems on simple, connected, undirected, weighted, and finite graphs is the first step in research on the same kind of problems on locally finite graphs, which can be used as a discrete analog of some continues Laplacian related problems. This analogy can be used to approximate these continuous problems by the family of such discrete ones.

The main goal is to obtain the existence and uniqueness of solution for problems connected with the discrete $s(x, \cdot)$-Laplacian on simple, connected, undirected, weighted, and finite graphs where nonlinearities are given in a non-potential form, with the minimal possible assumptions, positive solutions are also considered.

The non-classical monotonicity methods (see [1]) have been used to prove the existence and uniqueness of the solution. That can not be done for non-potential problems with the use of the classical variational approach. Assumption of non-empty graph boundary (see. [3]) was completely removed. Positive solutions results have been easily adapted to the considered setting.

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Cicles of corruption and democracy

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Presentation type: Contributed Talk

In this paper we propose a game theoretic model with three populations, namely a government, officials who serve the state, and citizens, to analyse the evolution of corruption in a society. The influence of democracy in corruption is modelled through the action of the citizens who exercise influence in the government because of their elective power since corruption causes a great displeasure in the citizens which can result in a vote against a ruler elite that promotes or is an accomplice to corruption. When immersed in a society in which corruption is a common occurrence, citizens may behave in a complacent manner with corruption because of a lack of valid alternatives to this behaviour even if they oppose corruption. Indeed, this complacent behaviour may also be observed in democratic societies and can lead to periods of growing and diminishing corruption. We are thus able to get a better understanding of some causes for the evolution of corruption and how the evolution may be halted and the effects of democracy and influence in this.

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Linearized stability in the context of an example by Rodrigues and Solà-Morales

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Presentation type: Contributed Talk

In a recent paper, Rodrigues and Solà-Morales construct an example of a continuously Fréchet differentiable discrete dynamical system in a separable Hilbert space for which the origin is an exponentially asymptotically stable fixed point, although its derivative at 0 has spectral radius greater than one. For maps on general Banach spaces we demonstrate that the slightly stronger, but also widely used concept of exponential stability allows a complete characterization in terms of the spectral radius. Moreover, under a spectral gap condition valid for compact and finite-dimensional linearizations these two stability notions are shown to be equivalent.

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Neutral Volterra Difference Equations of Advanced Type

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Presentation type: Contributed Talk

The inversion of a perturbed difference operator may yield the sum of a contraction and a compact operator. In this talk, we consider a neutral difference equation, we add and subtract a linear term and do the proper inversion to get what we call, Neutral Volterra Difference Equations of Advanced Type. Then we use Krasnoselskii fixed point theorem to study existence of solutions.

Stabilization of multiple equilibria with stochastic prediction-based control

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Presentation type: Contributed Talk

Prediction-Based control is applied to stabilize multiple equilibria of the continuous but, generally, non-smooth maps. Sufficient conditions of global stabilization are obtained. Introduction of noise allows to lower the level of average control. Theoretical results are illustrated by computer simulations.

Comparison of Tests for Oscillations in Delay/Advanced Difference Equations with Continuous Time

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Presentation type: Contributed Talk

In this paper we compare sufficient conditions for the oscillation of all solutions of the delay (advanced) difference equation with continuous time inspired by our results published in [Filomat **34(8)** (2020), 2693-2704] to relevant results in the literature. We provide various examples with constant delays (advances), but with variable or constant coefficients.

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Period-doubling in Bifurcation Diagrams from Dynamics of Some Special Kinds of Families of Transcendental Functions

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Presentation type: Contributed Talk

Over recent years, bifurcation, chaos and fractals are new scientific tools developed for solving more advanced linear and nonlinear systems. Due to wide-ranging applications, these are increasingly being involved in all areas of scientific research. Generally, the real issues and problems and issues of modern scientific, engineering, technological and economical researches are nonlinear in nature. Most nonlinear systems are extremely difficult to solve analytically or much harder to analyze. In a nonlinear system, a small change in a parameter can lead to sudden and dramatic changes in both the qualitative and quantitative behaviour of the system. During the last three decades of the 20th century, the excessive studies of nonlinear dynamics showing chaotic behaviour by using period-doubling in bifurcation diagrams of dynamical systems. It can visualize effectively through fractals. The purpose of this work provides an overview of fractals and chaos as well as explore period-doubling in bifurcation diagrams from dynamics of some special kinds of families of transcendental functions.

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Admissibility Criteria for Exponential Dichotomy of Discrete Nonautonomous Systems and Applications

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Presentation type: Contributed Talk

We present general input-output criteria for exponential dichotomy of discrete nonautonomous systems on the whole line, in the uniform case, in infinite dimensional spaces. We discuss the structure of the admissible pairs of sequence spaces and the connections between discrete admissibility and exponential dichotomy. Next, we present a new method of exploring the robustness of exponential dichotomy, that combines arguments from operator theory with control techniques and we give answers to two open problems regarding upper and lower bounds for the dichotomy radius, that were brought into attention on the occasion of the *25th International Conference on Difference Equations and Applications* held at University College London in 2019. Finally, we describe a general scheme for studying the robustness of exponential dichotomies in the uniform case and discuss some future directions.

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φ-Contractive parent-child infinite IFSs

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Presentation type: Contributed Talk

In this paper we introduce the notion of φ-contractive parent-child infinite iterated function system (pcIIFS) and we prove that the corresponding fractal operator is weakly Picard. The corresponding notions of shift space, canonical projection and their properties are also treated.

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Admissibility and μ-dichotomies for discrete dynamics

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Presentation type: Contributed Talk

Let $(X, \|\cdot\|)$ be a Banach space and $(A_n)_{n\in\mathbb{N}}$ be a sequence of bounded linear operators acting on X. Denote the *discrete evolution family* associated with the sequence $(A_n)_{n\in\mathbb{N}}$ by $\mathcal{A} = (\mathcal{A}_{m,n})_{m\geq n}$: $A_{m,n} = A_{m-1}\cdots A_n$, for $m > n$, and $A_{n,n} = \mathrm{Id}$.

Fix a sequence of nonnegative numbers $(\mu_m)_{m\in\mathbb{N}}$, strictly increasing and satisfying $\lim_{m\to+\infty}\mu_m = +\infty$, and let $(\|\cdot\|_m)_{m\in\mathbb{N}}$ be a sequence of norms in X such that, for each fixed m, the norm $\|\cdot\|_m$ is equivalent to $\|\cdot\|$. We say that the sequence of linear operators $(A_m)_{m\in\mathbb{N}}$ (or alternatively that the linear difference equation $x_{m+1} = A_m x_m,\ m \in \mathbb{N}$) admits a *$\mu$-dichotomy with respect to the sequence of norms* if there are projections P_m, $m \in \mathbb{N}$, such that $A_m|\ker P_m \to \ker P_{m+1}$ is invertible, $P_m\mathcal{A}_{m,n} = \mathcal{A}_{m,n}P_n$, $m, n \in \mathbb{N}$, and there are constants $\lambda, D > 0$ such that, for every $x \in X$ and $n, m \in \mathbb{N}$, we have

$$\|\mathcal{A}_{m,n}P_n x\|_m \leq D\,(\mu_m/\mu_n)^{-\lambda}\,\|x\|_n, \quad \text{for } m \geq n$$

and

$$\|\mathcal{A}_{m,n}Q_n x\|_m \leq D\,(\mu_n/\mu_m)^{-\lambda}\,\|x\|_n, \quad \text{for } m \leq n,$$

where $Q_m = \mathrm{Id} - P_m$ is the complementary projection and, for $m \leq n$, we use the notation $\mathcal{A}_{m,n} = (\mathcal{A}_{n,m})^{-1} : \ker P_n \to \ker P_m$.

In this talk we obtain characterizations of μ-dichotomies based on admissibility conditions. Additionally, we use the obtained characterizations to derive robustness results for the considered dichotomies. As particular cases, we recover several results in the literature concerning nonuniform exponential dichotomies and nonuniform polynomial dichotomies [1, 2] as well as new results for nonuniform dichotomies with logarithmic growth. This talk is based on [3].

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Milnor attractors in a family of nonautonomous dynamical systems generated by flat-topped tent maps

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Presentation type: Contributed Talk

If a dynamic process is generated by a one-dimensional map, then the insertion of a flat segment on the map will often lead to a superstable periodic orbit. This mechanism has been widely used in the control of chaos on one-dimensional systems in areas as diverse as cardiac dynamics (see [2]), telecommunications or electronic circuits (see [5] and references therein). Let $T : [-1, 1] \to [-1, 1]$ be the tent map and $T_u : [-1, 1] \to [-1, 1]$, $u \in [-1, 1]$, be the flat-topped tent map with constant value u in the plateau. In this work we consider families of nonautonomous dynamical systems $x_{n+1} = T_{(u,s_n)}(x_n)$ where $s \in \{0, 1\}^{\mathbb{N}_0}$ is the iteration pattern, $T_{(u,s_n)}(x) = T_u(x)$ if $s_n = 0$ and $T_{(u,s_n)}(x) = T(x)$ if $s_n = 1$. We consider as parameters the pairs $(u, s) \in [-1, 1] \times \{0, 1\}^{\mathbb{N}_0}$ and study the existence of a nonautonomous version of Milnor attractors and their coexistence with other kinds of nonautonomous attractors.

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A SHADOWING TYPE RESULT FOR THE DIFFERENCE EQUATIONS

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Presentation type: Contributed Talk

A difference equation is said to have the ***shadowing property*** if in a vicinity of every approximate solution there exists an exact solution of the difference equation.

In this talk, we will discuss the shadowing property for a class of semilinear difference equations of the form

$$x_{n+1} = A_n x_n + f_n(x_n) \quad n \in \mathbb{Z}, \tag{1}$$

where A_n, $n \in \mathbb{Z}$ are bounded, linear and invertible operators on a Banach space X and $f_n \colon X \to X$, $n \in \mathbb{Z}$ is a sequence of Lipschitz maps. In particular, we will formulate conditions under which (1) exhibits the shadowing property, which unlike some previous works (see [1, 2]) don't require any information related to the asymptotic behaviour of the linear part.

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A Note Regarding Extensions of Fixed Point Theorems via an Analysis of Iterated Functions

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Presentation type: Contributed Talk

The purpose of this work is to advance the current state of mathematical knowledge regarding fixed point theorems of functions. Such ideas have historically enjoyed many applications , for example, to the qualitative and quantitative understanding of differential, difference and integral equations. Herein, we extend an established result due to Rus [Studia Univ. Babeş-Bolyai Math., 22, 1977, 40-42] that involves two metrics to ensure wider classes of functions admit a unique fixed point. In contrast to the literature, a key strategy herein involves placing assumptions on the iterations of the function under consideration, rather than on the function itself. In taking this approach we form new advances in fixed point theory under two metrics and establish interesting connections between previously distinct theorems, including those of Rus [Studia Univ. Babeş-Bolyai Math., 22, 1977, 40-42], Caccioppoli [Rend. Acad. Naz. Linzei. 11, 1930, 31-49] and Bryant [The American Mathematical Monthly, 75, 1968, 399-400]. Our results make progress towards a fuller theory of fixed points of functions under two metrics. Our work lays the foundations for others to potentially explore applications of our new results to form existence and uniqueness of solutions to boundary value problems, integral equations and initial value problems.

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Strong convergence of adaptive Milstein methods for SDEs

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Presentation type: Contributed Talk

We introduce explicit adaptive Milstein methods for stochastic differential equations (SDEs) with no commutativity condition. The twice continuously differentiable drift and diffusion are separately locally Lipschitz and together satisfy a monotone condition. These methods rely on a class of path-bounded timestepping strategies which work by reducing the stepsize as solutions approach the boundary of a sphere, invoking a backstop method in the event that the timestep becomes too small. We prove that such schemes are strongly L_2 convergent of order one. This convergence order is inherited by an explicit adaptive Euler-Maruyama scheme in the additive noise case. Moreover we show that the probability of using the backstop method at any step can be made arbitrarily small. We compare our method to other fixed-step Milstein variants on a range of test problems.

On Distribution of Focal Points for Conjoined Bases of Symplectic Difference Systems

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Presentation type: Contributed Talk

We discuss a topic from the oscillation theory of symplectic difference systems. These systems include as special cases several important linear difference equations and systems, such as linear Hamiltonian difference systems, even order Sturm–Liouville difference equations, second order matrix Sturm–Liouville difference equations, or symmetric three-term matrix recurrence equations. In this talk we present a method for constructing a conjoined basis of the symplectic difference system having prescribed numbers of forward and backward focal points in a given bounded interval. This utilizes the theory of comparative index and a recent result by the authors on linear Hamiltonian differential systems.

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Nonlinear Time-Varying Delay Dynamic Equations on Time Scales with Impulses

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Presentation type: 'Contributed Talk'

The dynamic equations on time scales can model many real-world phenomena that involve discrete data, continuous data, or discrete-continuous data simultaneously. Differential equations with impulses arise in the mathematical modelling of several evolutionary processes that involve abrupt changes at certain moments. Also, in most real-world phenomena, the present state of a system depends on some previous history. Therefore, it is reasonable to include a time-delay term in the process. This talk aims to present some qualitative results concerning a new class of time-varying delay dynamic equations with impulses. We establish some criteria for the existence, uniqueness, and stability of the solution. Our approach is based on the results of fixed point theory and dynamic inequalities. To overcome the difficulties in establishing the uniqueness of the solution, we pose certain conditions on the time scale domain. We shall try to provide illustrative examples to support the results obtained.

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The roles that shooting methods can play in the theory of discrete boundary value problems

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Presentation type: Contributed Talk

For over 50 years, shooting methods have helped the scientific community to see through some of the fog associated with the qualitative and quantitative properties of solutions to ordinary differential equations [2]. However, the roles of shooting methods in the study of boundary value problems (BVPs) involving *difference equations* is yet to be fully understood. This sheltered state may have more to do with the human tendancy to focalize attention on the things that we have been conditioned to [1, p.v], and we have mostly percieved shooting methods only within the domain of differential equations. However, developing alternative perspectives in mathematics is important because they can open up new ways of thinking and working [3, p.1292], [4, Sec.3].

The purpose of this work is to move towards a more complete understanding of the roles that shooting methods can play in the theory of discrete BVPs. My position is that shooting methods can provide an important function in exploring discrete BVPs due to: their compatible characteristics; their accessibility; and the significance and flexibility of their conclusions. My position is realized through the establishment of new theory and is supported via exemplification.

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Analytical and numerical computation of the box dimension of trajectories for a class of systems having degenerate foci

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Presentation type: Contributed Talk

We study a class of polynomial planar systems with singularity of degenerate focus type without characteristic directions, given by

$$\begin{aligned}\dot{x} &= -ny^{2n-1} \pm nx^m y^{n-1}(x^{2m}+y^{2n})^k \\ \dot{y} &= mx^{2m-1} \pm mx^{m-1}y^n(x^{2m}+y^{2n})^k,\end{aligned}$$

where $m, n, k \in \mathbb{N}$. In the case where $m = n$ and m is odd, we analytically compute the box dimension of trajectory Γ to be

$$\dim_B \Gamma = 2 - \frac{2}{1+2kn}.$$

We also show the connection of box dimension to cyclicity of the system under a perturbation.

Furthermore, we develop an efficient numerical scheme for computation of the box dimension of trajectories of our system. The scheme validates our analytical result in the case $m = n$ and complements it in the case $m \neq n$. Our numerical scheme converges faster than naive methods for numerical box dimension computation and is optimized for spiral trajectories in our system.

This is a joint work with Renato Huzak, Darko Žubrinić and Vesna Županović.

Dynamical behavior of the stochastic Hepatitis C model

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Presentation type: Contributed Talk

In this paper we consider the dynamical behavior of stochastic Hepatitis C model with an isolation stage. We construct stochastic model on the basis of the deterministic one by incorporating randomness of the Gaussian white noise type. On that way we obtain five-stage system of stochastic differential equations. For our model we prove existence and uniqueness of the global positive solution and then we consider long time behavior of the solution. More precisely, we obtain the conditions for model parameters under which the disease goes to extinction, as well as ones under which we can claim persistence of the disease in population. Finally, we give a real life example to illustrate acquired theoretical results.

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On Topological Distributional Chaos

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Presentation type: Contributed Talk

The notion of chaos based on the probabilistic measure of upper and lower densities of the rate of proximality of pairs is termed as distributional chaos and was introduced by Schweizer and Smítal for continuous self-maps on compact metric spaces [1]. Since then the distributional chaos has evolved into three variants popularly known as $DC1$, $DC2$ and $DC3$. Shah et. al. extended the notion of distributional chaos for continuous self-maps on uniform spaces which are not necessarily compact and metrizable [2]. In this talk, we will discuss the relation between topological definitions of specification property and distributional chaos defined for uniformly continuous self maps on uniform spaces.

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Generalized nonautonomos G-dynamical systems

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Presentation type: Contributed Talk

Let X be a nonempty set and G be a nonempty subset of the power set of X denoted by $exp(X)$such that G containd $\emptyset$ and it is closed under arbitrary unions, then G is called a generalized topology (GT as acronym) on X. The pair (X, G) is called a generalized topological space (GTS as acronym).

Let $(X, d_n),\ n \in \mathbb{N}$ be a **compact** metric space and $f_n : (X, d_n) \to (X, d_{n+1})\ n \in \mathbb{N}$ be a continuous map. Following S. Kolyada and L. Snoha 1996, by a generalized non-autonomous dynamical system on X (GNDS), we will mean the sequence of functions $f_{1,\infty} = \{f_i\}_{i=1}^{\infty}$. If $f_i = f$ and $d_i = d$ for $i \in \mathbb{N}$, then the system is called autonomous and denote it by (f). The identity map on X will be denoted by Id. For any $i \in \mathbb{N}$, let $f_0^i = Id$ and for any $i \in \mathbb{N}$, let $f_i^n = f_{i+(n-1)} \circ \cdots \circ f_{i+1} \circ f_i$,

$f_i^{-n} = (f^n)^{-1} = f_i^{-1} \circ f_{i+1}^{-1} \circ \cdots \circ f_{i+(n-1)}^{-1}$.

In [6], the generalized entropy in generalized topological spaces is introduced and studied. In this paper, we introduce and study GT on nonautonomous G-dynamical systems.

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Fractal analysis of planar nilpotent singularities and numerical applications

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Presentation type: Contributed Talk

The goal of our work is to give a complete fractal classification of planar analytic nilpotent singularities. For the classification, we use the notion of box dimension of (two-dimensional) orbits on separatrices generated by the unit time map. We also show how the box dimension of the one-dimensional orbit generated by the Poincaré map, defined on the characteristic curve near the nilpotent center/focus, reveals an upper bound for the number of limit cycles near the singularity. We introduce simple formulas for numerical calculation of the box dimension of one- and two-dimensional orbits and apply them to nilpotent singularities.

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